APPM 1350 **Exam 3** **Summer 2021**

On the front page please write your name and clearly label each problem This exam is worth 100 points and has 5 questions on both sides of this paper.

- Make sure all of your work is on separate sheets of paper. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted, except for a computer for proctoring through Zoom.

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1. (15 pts) In your own words, answer the following concerning some of the fundamental mathematical concepts in Unit 3. Two or three sentences or a sentence and a brief sketch should be sufficient for each of your responses.
   (a) Suppose $\int_{-5}^{5} K(x)dx = 0$ for some continuous function, $K(x)$. Speculate on a possible value(s) for $\int_{-5}^{5} L(x)dx$, where $L(x) = (K(x))^2 + 2$. Explain why your speculation is sound.
   (b) The area under $g(x) = -x^2 + 16$ is approximated as the sum of four, equal-width rectangles drawn beneath the curve using the left-endpoint method on the interval $[0, 4]$. Explain whether this approximation will underestimate or overestimate the true value of the area beneath $g(x)$.
   (c) Consider a scientist using the Newton-Raphson method to find the roots of the function $h(x) = \frac{1}{3}x^3 - 4x - 3$. Explain why an initial guess of either $x = 2$ or $x = -2$ will not result in a convergent solution.

2. (20 pts) For each of the following, find f, the most general antiderivative.
   (a) $f'(\phi) = \phi - \sin(\phi) - \frac{1}{\pi}$.
   (b) $f''(z) = z^3 - \frac{1}{\sqrt{z}} + 12$.

3. (20 pts) For each of the following functions, find the requested derivative.
   (a) $W''(x)$ where $W(x) = \int_1^x \frac{\sin(z)}{z} dz$.
   (b) $B'(r)$ where $B(r) = \int_{-r^2}^{r^2} (9 - p^4)dp$.

$\boxed{\text{TURN OVER—More problems on the back!}}$

4. (25 pts) A subsurface oil storage tank suffers a breach in its protective clay lining. Oil is now leaking from the buried tank into the surrounding subsurface at a rate described by the function $R(t) = -t^2 + 10t + 3$, where t is measuree in days and $R(t)$ is measured in L/day. The function $R(t)$ has been estimated using a model fit to data taken from the tank's depth gauge every 12 hours (Fig 1: green points). The leak lasts for 5 days before it is instantly sealed and the leak stops. Environmental officials need to forecast the impact of this leak on the surrounding wildlife and have tasked you to estimate how much oil has leaked out.

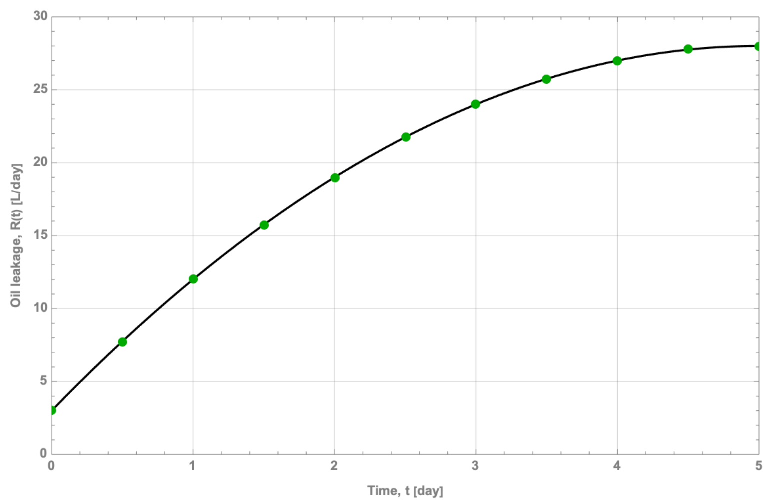

Figure 1: Leakage rate of subsurface oil tank. $R(t)$ has been fit to the depth gauge readings plotted as individual points.

(a) Set up a Riemann sum for $R(t)$ on the interval $[0, 5]$ using N equally spaced sub-intervals and right endpoints.

(b) Determine how much oil has leaked by simplifying your Riemann sum and evaluating it for the limit as $N \to \infty$.

(c) Compute the total amount of leaked oil by evaluating the definite integral of $R(t)$—verify your calculation matches the one you found in part (b).

5. (20 pts) A particle is constrained to move along the X-axis, subject to a force producing the following acceleration: $a(t) = -\frac{1}{4}t^2 + \cos(\frac{8}{\pi}t) + 10\ m/s^2$. The particle begins its motion at time $t = 0$ on the origin with an initial velocity of 12 m/s. For any time $t \geq 0$, determine the <u>exact</u> functions describing:

(a) the particle's velocity, $v(t)$;

(b) and the particle's position, $s(t)$.

Formulas: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$