On the front page please write your name and clearly label each problem This exam is worth 100 points and has 4 questions on both sides of this paper.

- Make sure all of your work is on separate sheets of paper. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted, except for a computer for proctoring through Zoom.

1. (15 pts) In your own words, answer the following concerning some of the fundamental mathematical concepts in Unit 2. Two or three sentences or a sentence and a brief sketch should be sufficient for each of your responses.
   
   (a) Support or refute the following: solving $y'(x) = 0$ will always produce the $x$ value which maximizes the function $y(x)$ on a given interval.
   
   (b) The function $g(x) = x^{2/5}$ is continuous and differentiable for all real numbers, excluding $x = 0$ where $g(x)$ is not differentiable due to the presence of a cusp. Does this mean the Mean Value Theorem can never be used in situations involving the function $g(x)$? Explain why or why not.
   
   (c) Consider: I just solved a related rates problem where a balloon was filling with air and I wanted to characterize the change in the balloon’s radius, $R$. My solution for the rate-of-change of the balloon’s radius was $dR/dt = -2.4 \text{ cm}^2/\text{min}$. Does my solution make sense in the context of the problem? Why or why not?

2. (20 pts) Determine the following derivatives for the provided functions.
   
   (a) $dz/dt$, for $z(t) = (t^3 - \cos(t^3))^3$.
   
   (b) $dy/dx$, for $y(x) = -7x^2y + \sin(x - y)$.

3. (30 pts) For this problem, let $f(x) = \frac{x}{(x^2 + 1)^{4/3}}$.
   
   (a) Find domain of $f(x)$ and any vertical or horizontal asymptotes, if they exist.
   
   (b) Determine the intervals over which $f(x)$ is increasing and decreasing and the extrema of this function.
   
   (c) Determine the intervals of concavity and the points of inflection for $f(x)$. You can borrow this expression for the second derivative, if you need: $f''(x) = \frac{8x(5x^2-9)}{9(x^2+1)^{10/3}}$.
   
   (d) Using the function’s characteristics you just found, sketch $f(x)$. Be sure to label the features of interest for the function that you found in parts (a)–(c).

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TURN OVER—More problems on the back!
4. (20 pts) The temperature of an ultra-cold gas in an experimental chamber is governed by the equation
\[ T(t) = -\frac{t^3}{3} + 3t^2 - 8t + 15, \]
where \( T \) and \( t \) are measured in K and hours, respectively. \( T(t) \) is defined for the interval \( 0 \leq t \leq 6. \)

(a) Using the Closed Interval Method, determine the absolute maximum and minimum temperatures achieved during this experiment and when they occurred.

(b) Suppose the energy contained by this ultra-cold gas, \( E(T) \), is determined for a given temperature by \( E(T) = kT^2 + E_0 \), where \( k \) and \( E_0 \) are gas-dependent constants and \( E(T) \) is measured in J. Determine the rate at which this gas’ energy is changing when the experiment reaches a time of \( t = 5 \) hours.

5. (15 pts) The intensity of optical radiation, \( I(\theta) \), measured by a polarimeter is governed by Malus’ Law,
\[ I(\theta) = I_0 \cos^2(\theta), \]
where \( I_0 \) is the intensity of the incoming radiation (in W/m\(^2\)) and \( \theta \) is the angle between the instrument’s polarizing screen and the vertical axis (in radians). In this type of instrument, the angle \( \theta \) ranges from 0 to \( \pi \). Suppose \( \theta \) is measured to be \( \pi/4 \), with a maximum possible error of \( \pi/30 \). For a radiation source of intensity \( I_0 = 1360 \) W/m\(^2\), determine the following approximations using differentials:

(a) The absolute error in \( I(\theta) \).

(b) The relative error in \( I(\theta) \).