**APPM 1350** **Exam 2** **Summer 2021**

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**On the front page please write your name and clearly label each problem** This exam is worth 100 points and has 4 questions on both sides of this paper.

- Make sure all of your work is on separate sheets of paper. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted, except for a computer for proctoring through Zoom.

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1. (15 pts) In your own words, answer the following concerning some of the fundamental mathematical concepts in Unit 2. Two or three sentences or a sentence and a brief sketch should be sufficient for each of your responses.

   (a) Support or refute the following: solving $y'(x) = 0$ will always produce the $x$ value which maximizes the function $y(x)$ on a given interval.

   (b) The function $g(x) = x^{2/5}$ is continuous and differentiable for all real numbers, excluding $x = 0$ where $g(x)$ is not differentiable due to the presence of a cusp. Does this mean the Mean Value Theorem can never be used in situations involving the function $g(x)$? Explain why or why not.

   (c) Consider: I just solved a related rates problem where a balloon was filling with air and I wanted to characterize the change in the ballon's radius, $R$. My solution for the rate-of-change of the ballon's radius was $dR/dt = -2.4\ \frac{cm^2}{min}$. Does my solution make sense in the context of the problem? Why or why not?

2. (20 pts) Determine the following derivatives for the provided functions.

   (a) $dz/dt$, for $z(t) = (t^3 - \cos(t^3))^3$.

   (b) $dy/dx$, for $y(x) = -7x^2y + \sin(x - y)$.

3. (30 pts) For this problem, let $f(x) = \dfrac{x}{(x^2+1)^{4/3}}$.

   (a) Find domain of $f(x)$ and any vertical or horizontal asymptotes, if they exist.

   (b) Determine the intervals over which $f(x)$ is increasing and decreasing and the extrema of this function.

   (c) Determine the intervals of concavity and the points of inflection for $f(x)$. *You can borrow this expression for the second derivative, if you need:* $f''(x) = \frac{8x(5x^2-9)}{9(x^2+1)^{10/3}}$.

   (d) Using the function's characteristics you just found, sketch $f(x)$. Be sure to label the features of interest for the function that you found in parts (a)–(c).

TURN OVER—More problems on the back!

4. (20 pts) The temperature of an ultra-cold gas in an experimental chamber is governed by the equation $T(t) = -t^3/3 + 3t^2 - 8t + 15$, where $T$ and $t$ are measured in K and hours, respectively. $T(t)$ is defined for the interval $0 \leq t \leq 6$.

   (a) Using the Closed Interval Method, determine the absolute maximum and minimum temperatures achieved during this experiment and when they occured.

   (b) Suppose the energy contained by this ultra-cold gas, $E(T)$, is determined for a given temperature by $E(T) = kT^2 + E_0$, where $k$ and $E_0$ are gas-dependent constants and $E(T)$ is measured in J. Determine the rate at which this gas' energy is changing when the experiment reaches a time of $t = 5$ hours.

5. (15 pts) The intensity of optical radiation, $I(\theta)$, measured by a polarimeter is governed by Malus' Law, $I(\theta) = I_0 \cos^2(\theta)$, where $I_0$ is the intensity of the incoming radiation (in $W/m^2$) and $\theta$ is the angle between the instrument's polarizing screen and the vertical axis (in radians). In this type of instrument, the angle $\theta$ ranges from $0$ to $\pi$. Suppose $\theta$ is measured to be $\pi/4$, with a maximum possible error of $\pi/30$. For a radiation source of intensity $I_0 = 1360\ W/m^2$, determine the following approximations using differentials :

   (a) The absolute error in $I(\theta)$.

   (b) The relative error in $I(\theta)$.