
On the front page please write your name and clearly label each problem This exam is worth 100 points and has 4 questions on both sides of this paper.

- Make sure all of your work is on separate sheets of paper. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted, except for a computer for proctoring through Zoom.

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1. (15 pts) In your own words, answer the following concerning some of the fundamental mathematical concepts in Unit 1. Two or three sentences or a sentence and a brief sketch should be sufficient for each of your responses.

   (a) Strictly speaking, $\frac{\pi}{4} \neq \frac{9\pi}{4}$. Explain why $\cos(\frac{\pi}{4}) = \cos(\frac{9\pi}{4})$.

   (b) What does it mean when a limit is "indeterminate?" How do we identify indeterminate expressions when calculating limits?

   (c) Support or refute the following claim: if a function is continuous over an interval it must be differentiable over that interval.

2. (20 pts) For this problem, let $f(x) = \dfrac{\cos(x)}{x^2 - \pi^2}$.

   (a) Determine the domain of $f(x)$. Express your answer in interval or set-builder notation.

   (b) Verify that $f(x)$ has exactly two vertical asymptotes and determine the equations describing them.

   (c) Justify that $f(x)$ has a single horizontal asymptote and determine the equation describing it.

3. (20 pts) The following two limits both lack the direct substitution property. For each, briefly explain why they lack this property and calculate the limits. If the limits do not exist, indicate this by writing "DNE."

   (a) $\displaystyle\lim_{t\to\infty} \sqrt{t^2 + 2t} - \sqrt{t^2 - 2t}$

   (b) $\displaystyle\lim_{\phi\to 0} \phi^2 \sin\left(\frac{1}{\phi}\right)$

TURN OVER—More problems on the back!

4. (15 pts) Let $g(x)$ be defined as,

$$g(x) = \begin{cases} c, & x = -1 \\ \frac{x^2-x-2}{x+1}, & x \neq -1 \end{cases},$$

where c is a yet-to-be-determined constant. Use the definition of continuity to answer the following:

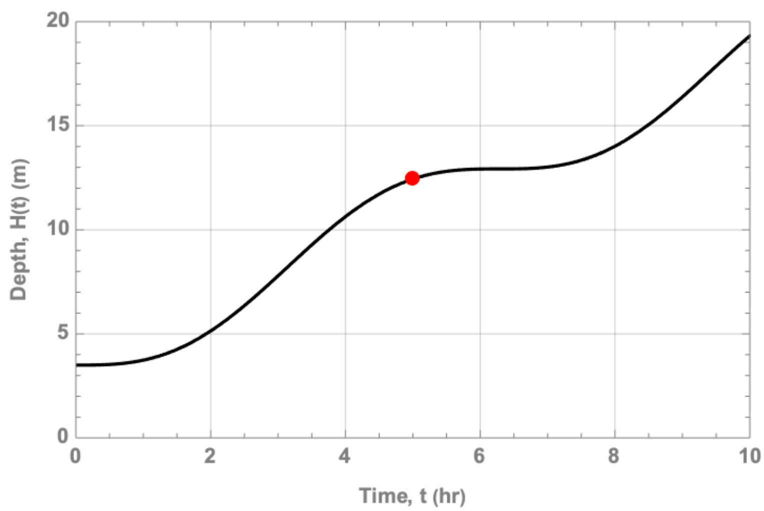
(a) What is the value of c that makes $g(x)$ continuous on $\mathbb{R}$?

(b) Why must $g(x)$ have a root on the interval $[0, 3]$?

5. (30 pts) Consider the function,

$$H(t) = \frac{3}{2}(t - \sin(t)) + \frac{7}{2},$$

which measures the depth of water in an emergency storage tank (in meters) as a function of time (in hours). The function is defined for the time interval of $[0, 10]$ hours. The tank is being filled from an external valve, however water is drawn from the tank for cooling purposes every few hours. A plot of $H(t)$ shown below:



(a) Categorize this function.

i. What kind of general function archetypes describe this (e.g., linear, polynomial, sinusoid, rational, etc.)?

ii. Is it even, odd, or neither?

iii. Over what intervals of time is this function increasing or decreasing? *If needed, you can approximate intervals of time using the plot.*

(b) Find the rate of change of $H(t)$ and describe what it specifically represents within the context of this problem.

(c) Determine the equation of the line tangent to $H(t)$ at the time $t = 5$ hours (red point on plot). *Leave trigonometric expressions like* $\sin(5)$ *as exact—do not try to approximate a decimal value.*