

**On the front page please write your name and clearly label each problem** This exam is worth 100 points and has 4 questions on both sides of this paper.

- Make sure all of your work is on separate sheets of paper. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted, except for a computer for proctoring through Zoom.
- You must use methods that we have learned in class thus far to solve the problems. (Dominance of powers is not valid)

1. (35 pts) Limits and Continuity (The following questions are unrelated)

(a) Find the following limits:

i.  $\lim_{t \rightarrow 0} \frac{\sqrt{1-t} - \sqrt{1+t}}{t}$

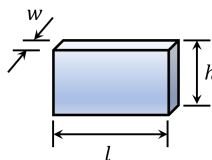
ii.  $\lim_{x \rightarrow \infty} e^{-x} \sin(x)$

iii.  $\lim_{\theta \rightarrow 0} \frac{\sin(\theta^3)}{\sin^3(2\theta)}$

(b) Does  $f(x) = x^3 - \cos(2x) + \sin(x)$  cross the x-axis on the interval  $[0, \pi]$ ? Explain.

2. (25 pts) Derivatives (The following questions are unrelated)

(a) Suppose that you are flying with an airline that requires that the sum of the dimensions of carry-on luggage ( $l + w + h$ ) must be no more than 90 cm and the length must be twice the height (see illustration).



What is the largest volume that such a carry-on can hold and the corresponding dimensions?

(b)  $\frac{e^{x/y}}{xy} = 1$ . Find  $\frac{dy}{dx}$ .

TURN OVER—More problems on the back!

3. (20 pts) Integrals (The following questions are unrelated)

(a) Integrate

$$\int_{\ln(2)}^{\ln(3)} \frac{e^{2x}}{e^{2x} - 1} dx$$

(b) Let the function  $g(x)$  be defined as

$$g(x) = \int_{\sqrt{x}}^1 \cos(t^2) dt$$

on the interval  $(0, 2\pi)$ . What are the intervals of increase and decrease? Justify.

4. (20 pts) Inverses, Logs, and Exponentials (The following questions are unrelated)

(a) Suppose  $f(x) = \frac{1}{1+x^2}$  on the interval  $(-\infty, 0]$ .

i. Find  $f^{-1}(x)$ .

ii. What is the domain and range of  $f^{-1}(x)$ ?

iii. Find  $(f^{-1})'(1/2)$  without finding the derivative of  $f^{-1}$ .

(b) Polonium-210 has a half-life of 138 days. How long would it take for a quantity of Polonium to decay to 1/10 of its original amount?

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