APPM 1350 **Exam 1** **Summer 2020**

On the front page please write your name and clearly label each problem This exam is worth 100 points and has 4 questions on both sides of this paper.

- Make sure all of your work is on separate sheets of paper. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted, except for a computer for proctoring through Zoom.

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1. (25 pts) For this problem, let $f(x) = \dfrac{\sin(x)}{x^2 - 1}$.
   (a) What is the domain of $f(x)$?
   (b) Does $f(x)$ have any horizontal asymptotes? If not, demonstrate this. If yes, determine the equation(s) of the horizontal asymptote(s).
   (c) Does $f(x)$ have any vertical asymptotes? If not, demonstrate this. If yes, determine the equation(s) of the vertical asymptote(s).

2. (30 pts) Calculate the following limits, if they exist. If a limit does not exist, indicate this by writing "DNE".
   (a) $\lim_{\theta \to 0} \theta \sin\left(\dfrac{1}{\theta}\right)$
   (b) $\lim_{x \to \infty} \sqrt{x^2 + 2x} - \sqrt{x^2 - 2x}$
   (c) Show that $\lim_{h \to 0} \dfrac{\cos(h) - 1}{h} = 0$. (Hint: multiply by the conjugate and use a trig identity)

3. (20 pts) The following questions are not related. Justify your answers and cite any theorems you use.
   (a) Let $f(x) = \begin{cases} c, & x = -1 \\ \frac{x^2 - x - 2}{x+1}, & x \neq -1 \end{cases}$. Use the definition of continuity to determine the value of c that makes $f(x)$ continuous on $\mathbb{R}$.
   (b) Given that f is a function where $f(0) = -1$ and $f(1) = 2$, is there a root in the interval $[0, 1]$? Explain why or why not (using theorems).

TURN OVER—More problems on the back!

4. (25 pts) The following questions are not related. Justify your answers and cite any theorems you use.

   (a) Find the equation of the tangent line to $f(x) = x^{1/3} + x$ at $x = 1$.

   (b) Use the definition of the derivative along with the angle addition formula $\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$ and the result from Problem 2c to find $f'(\theta)$ where $f(\theta) = \sin(\theta)$.
