
On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 150 points and has 4 questions.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
 - **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
 - Notes, papers, calculators, cell phones, and other electronic devices are not permitted.
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1. (43 pts total)

(a) Evaluate the following integrals:

i. (9 pts) $\int \frac{(1 + e^x)^2}{e^x} dx$

ii. (10 pts) $\int_0^{\frac{\pi}{2}} \frac{\sin(x)}{1 + \cos(x)} dx$

(b) Evaluate the following limits:

i. (8 pts) $\lim_{x \rightarrow -\infty} \tanh(2x)$

ii. (8 pts) $\lim_{x \rightarrow 0} x^2 \cos\left(\frac{1}{\sqrt{x}}\right)$

iii. (8 pts) $\lim_{x \rightarrow 0^-} e^{3/x}$

2. (22 pts total) Find the area of the largest rectangle inscribed in a right triangle with leg lengths 3 and 4 if two sides of the rectangle lie along the legs.

3. (40 pts total) The following problems are unrelated.

(a) (11 pts) Calculate y' if $y = \tan(xy)$.

(b) (13 pts) Find the equation of the tangent line to the curve of the function $f(x) = (\ln(x))^x$ at the point $(e, 1)$. Write your answer in the form $y = mx + b$. Use this linearization to estimate $\ln(3)^3$.

(c) (16 pts) Let $f(x) = \ln(2 + \ln(x))$. Determine the domain of f and f^{-1} , and find a formula for $f^{-1}(x)$.

4. (45 pts total) Let $g(x) = \int_{-x^3}^8 e^{t^2} dt$.

(a) (12 pts) Calculate $g(-2)$ and $g'(-2)$.

(b) (15 pts) Find the intervals on which the graph of $g(x)$ is concave up/down.

(c) (10 pts) Demonstrate that g is one-to-one.

(d) (8 pts) Calculate $(g^{-1})'(0)$.
