1. (43 pts total)
   (a) Evaluate the following integrals:
   \[ \int (1 + e^x)^2 dx \quad \text{(9 pts)} \]
   \[ \int_0^{\frac{\pi}{2}} \frac{\sin(x)}{1 + \cos(x)} dx \quad \text{(10 pts)} \]
   (b) Evaluate the following limits:
   \[ \lim_{x \to -\infty} \tanh(2x) \quad \text{(8 pts)} \]
   \[ \lim_{x \to 0} x^2 \cos \left( \frac{1}{\sqrt{x}} \right) \quad \text{(8 pts)} \]
   \[ \lim_{x \to 0^-} e^{3/x} \quad \text{(8 pts)} \]

2. (22 pts total) Find the area of the largest rectangle inscribed in a right triangle with leg lengths 3 and 4 if two sides of the rectangle lie along the legs.

3. (40 pts total) The following problems are unrelated.
   (a) (11 pts) Calculate \( y' \) if \( y = \tan(xy) \).
   (b) (13 pts) Find the equation of the tangent line to the curve of the function \( f(x) = (\ln(x))^x \) at the point \((e, 1)\). Write your answer in the form \( y = mx + b \). Use this linearization to estimate \( \ln(3)^3 \).
   (c) (16 pts) Let \( f(x) = \ln(2 + \ln(x)) \). Determine the domain of \( f \) and \( f^{-1} \), and find a formula for \( f^{-1}(x) \).

4. (45 pts total) Let \( g(x) = \int_{-x^3}^{8} e^{t^2} dt \).
   (a) (12 pts) Calculate \( g(-2) \) and \( g'(-2) \).
   (b) (15 pts) Find the intervals on which the graph of \( g(x) \) is concave up/down.
   (c) (10 pts) Demonstrate that \( g \) is one-to-one.
   (d) (8 pts) Calculate \( (g^{-1})'(0) \).