
On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 100 points and has 4 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

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1. Evaluate the following integrals:

   (a) (7 pts) $\displaystyle\int \frac{x^2}{(x^3+1)^2}\,dx$

   (b) (8 pts) $\displaystyle\int_0^2 x\sqrt{x+1}\,dx$

   (c) (7 pts) $\displaystyle\int \cos(x)\sin(x)\,dx$

   (d) (8 pts) $\displaystyle\int_0^{\pi/4} \tan(x)\sec(x)\,dx$

2. The following problems are unrelated. Fully justify your answers and cite any theorems you use.

   (a) (6 pts) Suppose we're using Newton's method to determine the root of the function $f(x) = x^3 - 2x - 5$. Use Newton's method to determine the second approximation x_2 of the root, given an initial guess of $x_1 = 2$.

   (b) (8 pts) Suppose that $f(x)$ is integrable on the interval $[1, 8]$. Suppose further that $\displaystyle\int_1^6 2f(x)dx = 10$ and $\displaystyle\int_6^8 f(x)dx = -2$. Evaluate $\displaystyle\int_1^8 3f(x)dx$.

   (c) (8 pts) Evaluate the following limit:

   $$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\left[\left(\frac{k}{n}\right)^3 + 1\right].$$

   (d) (6 pts) Show that the value of $\displaystyle\int_0^1 \sqrt{1+\cos(x)}dx$ cannot possibly be 2.

3. The velocity of a particle is given by $v(t) = t^2 - \sin(t)$ for $0 \le t \le \pi$.

   (a) (7 pts) Estimate the net displacement of the particle using a midpoint approximation with two equispaced subintervals.

   (b) (8 pts) Determine the average velocity of the particle.

   (c) (5 pts) Write (but do not evaluate) an integral to calculate the total distance traveled by the particle.

TURN OVER—More problems on the back!

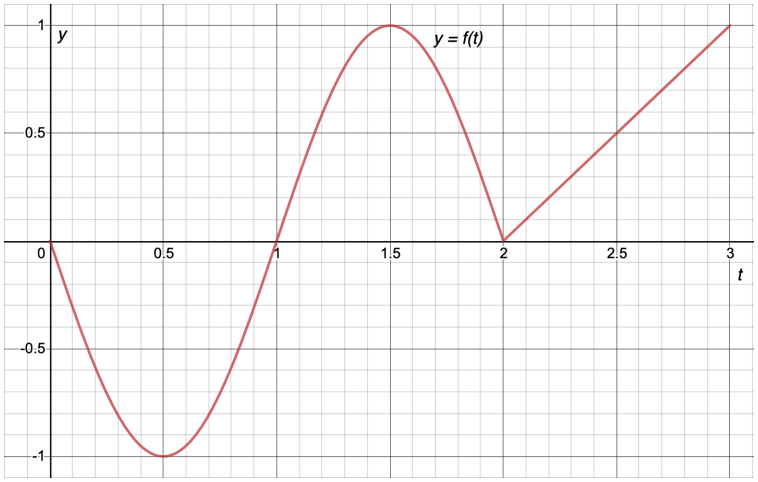

Figure 1: The graph of $y = f(t)$ for problem 4.

4. Figure 1 depicts the graph of a function $f(t)$, for $0 \leq t \leq 3$. For the following problems, define $g(x) = 2 + \int_{3/2}^{x} f(t)dt$.

   (a) (8 pts) Determine the values of $g(3/2)$, $g'(3/2)$, and $g''(3/2)$.

   (b) (7 pts) Determine the interval(s) on which $g(x)$ is increasing.

   (c) (7 pts) Determine the interval(s) on which $g(x)$ is concave up.

The following identities may be useful:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2.$$