On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 100 points and has 4 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

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1. (20 pts) For each of the following, find the requested information:

   (a) $y = \sec^2(3x)$; $\frac{dy}{dx}$.

   (b) $y = \dfrac{x}{\sqrt{2-x^2}}$; $\frac{dy}{dx}$.

   (c) $y^2(2-x) = x^3$; the slope of the tangent line to the curve at the point (1,1).

2. (24 pts) The following problems are unrelated. Fully justify your answers and cite any theorems you use.

   (a) A small child flies a kite at a height of 30 m. The wind carries the kite horizontally away from her at a rate of 6 m/s. How fast must she let out the string when the kite is 60 m away from her?

   (b) Find two positive numbers whose sum is 18 and whose product is as large as possible.

3. (28 pts) Let $f(x) = \dfrac{8}{x^2-4}$, with $f'(x) = -\dfrac{16x}{(x^2-4)^2}$ and $f''(x) = \dfrac{16(3x^2+4)}{(x^2-4)^3}$.

   (a) Find all asymptotes for f. Justify your answers using the appropriate limits.

   (b) Find the intervals on which f is increasing and decreasing, respectively. Justify your answers.

   (c) Find the local maximum and minimum values for the function f. Justify your answer.

   (d) Find the intervals of concavity and the points of inflection for the function f. Justify your answers.

   (e) Sketch the graph of f, using your results from (a)-(d) to inform your sketch. Label all asymptotes, local maxima/minima, and inflection points on the graph.

4. (28 pts) The following problems are unrelated. Fully justify your answers and cite any theorems you use.

   (a) Let $f(x) = (1+x)^k$ for some constant k. Determine the tangent line approximation to f at $(0,1)$. Use this to estimate $\sqrt{1.2}$ and $\sqrt[3]{1.3}$.

   (b) Consider the curve $y = \dfrac{x^2-3}{2x-4}$. Find the equation of the slant asymptote to the curve. Write down (**but do not evaluate**) the limit that you would use to demonstrate that the line you found is a slant asymptote.

   (c) Suppose that f is continuous and differentiable on the interval $[-1,4]$. Suppose further that $f(-1) = 1$ and $f'(x) \leq 4$. What is the largest possible value of $f(4)$?
