APPM 1350 **Exam 1** **Summer 2019**

On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 100 points and has 4 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

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1. (26 pts) For this problem, let $f(x) = \dfrac{x-3}{\sqrt{x}}$.
   (a) What is the domain of $f(x)$?
   (b) Where is $f(x)$ continuous?
   (c) Does $f(x)$ have any horizontal asymptotes? If not, demonstrate this. If yes, determine the equation(s) of the horizontal asymptote(s).
   (d) Does $f(x)$ have any vertical asymptotes? If not, demonstrate this. If yes, determine the equation(s) of the vertical asymptote(s).
   (e) Determine the equation of the tangent line to the curve $y = f(x)$ at the point $(1, -2)$.

2. (28 pts) Calculate the following limits, if they exist. If a limit does not exist, indicate this by writing "DNE".
   (a) $\lim_{x\to 0} \sin(x + \cos(x))$
   (b) $\lim_{x\to 3} \left[\dfrac{1}{x-3} - \dfrac{5}{x^2-x-6}\right]$
   (c) $\lim_{x\to 0} x^2 \sin\left(\cos\left(\dfrac{1}{x}\right)\right)$
   (d) $\lim_{x\to 0^-} \dfrac{x}{|x|}$

3. (26 pts) The following questions are not related. Justify your answers and cite any theorems you use.
   (a) Let $r(s) = \begin{cases} as^2 + 9, & \text{if } s \le 2 \\ \dfrac{s^2+s-6}{s-2}, & \text{if } s > 2. \end{cases}$ Use the *definition of continuity* to determine the value of a that makes $r(s)$ continuous on $\mathbb{R}$.
   (b) Suppose that a continuous function $f(x)$ satisfies $0 < f(x) < 1$ for every x in $[0, 1]$. Consider $g(x) = f(x) - x$; demonstrate that $g(x)$ has at least one root in the interval $(0, 1)$. Remember to cite any theorems that you may use. (*Hint*: Note that g is also continuous).
   (c) Consider $g(t) = t^2 - 3t + 2$. Use the *limit definition of the derivative* to calculate $g'(2)$.
   (d) Consider $h(x) = \dfrac{\tan(x)}{x^3}$. Is $h(x)$ odd, even, or neither? Justify your answer.

TURN OVER—More problems on the back!

4. (20 pts) For each of the following statements, determine if the statement is **TRUE** or **FALSE**. If the statement is **TRUE**, briefly explain why. If the statement is **FALSE**, briefly explain why or provide an example that shows the statement is false.

   (a) If a differentiable function $f(x)$ is periodic with period T, then $f'(x)$ is also periodic with period T. (Reminder: f is periodic with period T if $f(x+T) = f(x)$ for each x in its domain.)

   (b) If $\lim_{x\to\infty} f(x) = L$, then L is not in the range of f.

   (c) The curve $y = \frac{1}{x-1}$ has a vertical asymptote given by the line $y = 1$.

   (d) If $f'(a)$ exists, then $\lim_{x\to a} f(x) = f(a)$.
