1. [30 pts] Let \( f(x) = \frac{e^x}{4 - e^x} \).
   (a) Find all asymptotes, if any, of the graph of \( f(x) \). Full justification requires the appropriate use of limits.
   (b) Find the linearization of \( f(x) \) at \( x = 0 \).
   (c) Find the area under the graph of \( f(x) \) between \( x = 0 \) and \( x = 1 \).

2. [35 pts] Consider the function \( g(x) = \int_{-1}^{2x} \sqrt{1 + \sin t} \, dt \)
   (a) [5 pts] Find \( g(-1/2) \).
   (b) [10 pts] Will \( g(x) \) attain a maximum value on the interval \([0, \pi/2]\)? Explain briefly.
   (c) [10 pts] Let \( c \) be the smallest positive critical number of \( g(x) \). Find \( c \) and determine, with justification, whether or not \( g(c) \) is a relative extremum.
   (d) [10 pts] Determine, with justification, if \( g(x) \) has an inflection point in the interval \((0, \pi/2)\).

3. [65 pts] The following problems are unrelated.
   (a) [10 pts] Find \( (f^{-1})'(2) \) where \( f(x) = x^3 + 2x - 1 \).
   (b) [6 pts] Let \( a > 0 \) be a constant. If \( x = a \sin \theta \), find \( \tan \theta \) in terms of \( x \) and \( a \).
   (c) [28 pts] Find the following limits.
      i. \( \lim_{x \to 0} (1 - 3x)^{1/x} \)  
      ii. \( \lim_{x \to \infty} \cos^{-1} \left( \frac{1 + x^2}{1 + 2x^2} \right) \)  
      iii. \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{i}{n^2} \)  
      iv. \( \lim_{x \to 1} \frac{\sqrt{x + 1} - \sqrt{2}}{x - 1} \)
   (d) [21 pts] Evaluate the following integrals.
      i. \( \int_{\sqrt[3]{3}}^{\sqrt[3]{1/3}} \frac{1}{1 + 9x^2} \, dx \)  
      ii. \( \int \frac{x}{\sqrt{5x + 1}} \, dx \)  
      iii. \( \int \frac{\sinh w}{1 + \sinh^2 w} \, dw \)

4. [10 pts] A bug crawls along the curve \( y = \cosh x \). If the \( y \)-coordinate of the bug is changing at a rate of \(-4 \) mm/sec, how fast is its \( x \)-coordinate changing when \( x = \ln 3 \)? Write your final answer without any hyperbolic functions, using appropriate definitions.

5. [10 pts] Suppose an experimental population of fruit flies in your lab grows at a rate that is proportional to the number of flies present. Your lab assistant tells you that there were 100 flies after 2 days and 300 flies after 4 days. Approximately how many flies were in the original population?

Potentially Helpful Formulas

\[
\sum_{i=1}^{n} i = \frac{n(n + 1)}{2} \quad \sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6} \quad \sum_{i=1}^{n} i^3 = \left[ \frac{n(n + 1)}{2} \right]^2
\]