APPM 1350 **Final Exam** **Summer 2018**

On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 150 points and has 5 questions.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted. **Please begin each problem on a new page.**
- You will be taking this exam in a proctored and honor code enforced environment. This means: no notes or papers, calculators, cell phones, or other electronic devices are permitted.

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1. [30 pts] Let $f(x) = \dfrac{e^x}{4 - e^x}$.
   (a) Find all asymptotes, if any, of the graph of $f(x)$. Full justification requires the appropriate use of limits.
   (b) Find the linearization of $f(x)$ at $x = 0$.
   (c) Find the area under the graph of $f(x)$ between $x = 0$ and $x = 1$.

2. [35 pts] Consider the function $g(x) = \displaystyle\int_{-1}^{2x} \sqrt{1 + \sin t}\, \mathrm{d}t$
   (a) [5 pts] Find $g(-1/2)$.
   (b) [10 pts] Will $g(x)$ attain a maximum value on the interval $[0, \pi/2]$? Explain briefly.
   (c) [10 pts] Let c be the smallest positive critical number of $g(x)$. Find c and determine, with justification, whether or not $g(c)$ is a relative extremum.
   (d) [10 pts] Determine, with justification, if $g(x)$ has an inflection point in the interval $(0, \pi/2)$.

3. [65 pts] The following problems are unrelated.
   (a) [10 pts] Find $(f^{-1})'(2)$ where $f(x) = x^3 + 2x - 1$.
   (b) [6 pts] Let $a > 0$ be a constant. If $x = a \sin\theta$, find $\tan\theta$ in terms of x and a.
   (c) [28 pts] Find the following limits.

   i. $\displaystyle\lim_{x\to 0} (1 - 3x)^{1/x}$ ii. $\displaystyle\lim_{x\to\infty} \cos^{-1}\left(\frac{1 + x^2}{1 + 2x^2}\right)$ iii. $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n^2}$ iv. $\displaystyle\lim_{x\to 1} \frac{\sqrt{x+1} - \sqrt{2}}{x - 1}$

   (d) [21 pts] Evaluate the following integrals.

   i. $\displaystyle\int_{1/3}^{\sqrt{3}/3} \frac{1}{1 + 9x^2}\, \mathrm{d}x$ ii. $\displaystyle\int \frac{x}{\sqrt{5x + 1}}\, \mathrm{d}x$ iii. $\displaystyle\int \frac{\sinh w}{1 + \sinh^2 w}\, \mathrm{d}w$

4. [10 pts] A bug crawls along the curve $y = \cosh x$. If the y–coordinate of the bug is changing at a rate of -4 mm/sec, how fast is its x–coordinate changing when $x = \ln 3$? Write your final answer without any hyperbolic functions, using appropriate definitions.

5. [10 pts] Suppose an experimental population of fruit flies in your lab grows at a rate that is proportional to the number of flies present. Your lab assistant tells you that there were 100 flies after 2 days and 300 flies after 4 days. Approximately how many flies were in the original population?

POTENTIALLY HELPFUL FORMULAS

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$