APPM 1350 **Exam 3** **Summer 2018**

On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 5 questions.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted. **Please begin each problem on a new page.**
- You will be taking this exam in a proctored and honor code enforced environment. This means: no notes or papers, calculators, cell phones, or other electronic devices are permitted.

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1. [35 pts] Consider the function $f(x) = x^4 - 6x^2$.

   (a) [2 pts] What is the domain of $f(x)$?

   (b) [3 pts] Is $f(x)$ odd, even or neither? Justify your answer algebraically.

   (c) [3 pts] Find all the x–intercepts and y–intercepts of $f(x)$, if any.

   (d) [2 pts] Find all the asymptotes of $f(x)$, if any.

   (e) [2 pts] Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

   (f) [2 pts] Find $f'(x)$. Check your answer very carefully before proceeding.

   (g) [2 pts] Find $f''(x)$. Check your answer very carefully before proceeding.

   (h) [3 pts] Where is $f(x)$ increasing and where is $f(x)$ decreasing? Write your answer using interval notation.

   (i) [3 pts] Find all local extrema of $f(x)$, if it possesses any.

   (j) [3 pts] Where is $f(x)$ concave up and where is $f(x)$ concave down? Write your answer using interval notation.

   (k) [3 pts] Find all inflection points of $f(x)$, if it possesses any.

   (l) [7 pts] Sketch the graph of $f(x)$. Label all intercepts, relative extrema (if any) and inflection points (if any).

2. [18 pts] Consider the function $p(x) = x^3 - 3x + 4$.

   (a) Write the equation that uses Newton's method to find the root(s) of $p(x)$.

   (b) Using your answer to part (a), if $x_1 = 0$, find x_2.

   (c) Suppose x_1 is chosen such that $x_2 = -1$. What is x_3 in this case? Explain briefly.

3. [15 pts] The following problems are unrelated.

   (a) If $f(x) = 10 - x^2$, $1 \le x \le 5$, evaluate the Riemann sum with $n = 4$, taking the sample points to be right endpoints.

   (b) Evaluate $\int_2^{10} |x - 7| \, \mathrm{d}x$ by interpreting it in terms of areas.

   (c) Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \left(\frac{i}{n}\right)^2$.

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4. [12 pts] If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box. Be sure to show that this is the maximum volume.

5. [20 pts] The following problems are unrelated.

   (a) [6 pts] Let $g(x) = \dfrac{x}{x^2+1}$. Noting that $g'(x) = \dfrac{1-x^2}{(x^2+1)^2}$ and $g''(x) = \dfrac{2x\left(x^2-3\right)}{\left(x^2+1\right)^3}$, use the **second** derivative test to classify the relative extrema of $g(x)$.

   (b) [6 pts] Find the most general antiderivative of $f(w) = \dfrac{5}{\sqrt[3]{w^2}} + \dfrac{2}{\sqrt{w^3}}$.

   (c) [8 pts] A particle is moving with an acceleration $a(t) = 10\sin t + 3\cos t$. When time $t = 0$, the particle's position is 0 and when $t = 2\pi$ its position is 12. Find its position when $t = 5\pi/2$.

POTENTIALLY HELPFUL FORMULAS

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$