

On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 6 questions.

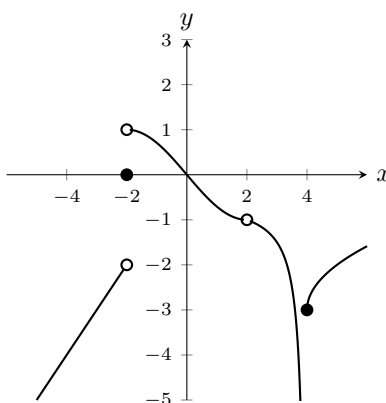
- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted. **Please begin each problem on a new page.**
- You will be taking this exam in a proctored and honor code enforced environment. This means: no notes or papers, calculators, cell phones, or other electronic devices are permitted.

1. Let $f(x) = 2 + \frac{2 - 2x}{x^2 - 5x + 4}$

- (a) [6 pts] What is the domain of f ? Write your answer using interval notation.
- (b) [9 pts] Does f have any removable discontinuities? Justify your answer using limits.
- (c) [9 pts] Find the asymptotes of f . Justify your answer using limits.

2. [16 pts] Using the graph of $f(x)$ in the figure below, compute the following:

- a. $\lim_{x \rightarrow -2^-} f(x)$
- b. $\lim_{x \rightarrow -2^+} f(x)$
- c. $\lim_{x \rightarrow -2} f(x)$
- d. $\lim_{x \rightarrow 2} f(x)$
- e. $f(2)$
- f. $\lim_{x \rightarrow 4^-} f(x)$
- g. $\lim_{x \rightarrow 4^+} f(x)$
- h. $\lim_{x \rightarrow 4} f(x)$



- 3. (a) [6 pts] What three conditions must be met for a function $f(x)$ to be continuous at the point a ?
- (b) [18 pts] Determine where the following functions are continuous, writing your answer using interval notation.
 - i. $f(x) = \cos(\sin(\sqrt{x})) - (x^4 - x^2 + 3)$
 - ii. $f(x) = \frac{|x - 5|}{x - 5}$
 - iii. $f(x) = \begin{cases} \frac{\cos 3x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

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4. [10 pts] The following problems are not related.

(a) Is there a value of x such that $x^2 - \sqrt{x-1}$ equals 4? Justify your answer.

(b) Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin 3x}{x} + \frac{6x-9}{x^3-12x+3} \right)$.

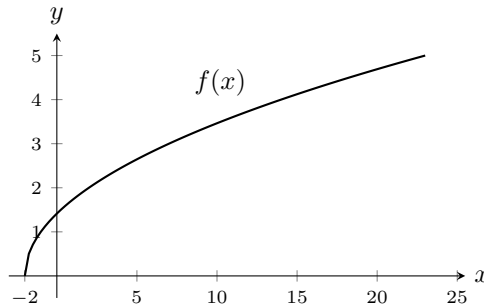
5. Let $f(x) = \sqrt{x+2}$. If you need to compute any derivatives, you must use the definition.

(a) [5 pts] Find the average rate of change of f over the interval $[7, 14]$. Simplify your answer. What geometric property of the graph of $f(x)$ does this average rate of change represent?

(b) [6 pts] Find the instantaneous rate of change of f at $x = 2$. What geometric property of the graph of $f(x)$ does this instantaneous rate of change represent?

(c) [6 pts] Find the slope/intercept form of the tangent line to the graph of $y = f(x)$ at the point $x = 2$.

(d) [4 pts] The graph of $f(x)$ is shown in the figure below. In your bluebook, sketch a graph of $f'(x)$.



6. [5 pts] In your bluebook, write **TRUE** if the statement is true and write **FALSE** if the statement is false. No justification required and no partial credit given.

(a) $\cos 2x = 2$ has no solutions.

(b) $f(x) = \sqrt{x^2 + x - 6}$ and $g(x) = \frac{1}{\sqrt{x^2 + x - 6}}$ have the same domain.

(c) If $f(-x) = -f(x)$ for all x in the domain of the function f , then the graph of $f(x)$ is symmetric with respect to the x -axis.

(d) If a function has a jump discontinuity at a point c in its domain, then the function is not differentiable at the point c .

(e) If $\lim_{x \rightarrow 5} f(x) = 0$ and $\lim_{x \rightarrow 5} g(x) = 0$, then $\lim_{x \rightarrow 5} \frac{f(x)}{g(x)}$ does not exist.