- 1. (28 points) The following problems are not related.
 - (a) Find the general antiderivative of $g(x) = \frac{e^{\sqrt{x}}}{\sqrt{x}}$.
 - (b) Use logarithmic differentiation to find the derivative of $y = (x^4 + 1)^x$. You do not need to simplify your answer.
 - (c) Find the derivative of $f(x) = \int_0^{\cos(x)} \sqrt{1+t^3} dt$.
- 2. (26 points) The following problems are not related:
 - (a) Find the derivative of $f(x) = \ln(\tan^{-1}(x))$.
 - (b) Evaluate the definite integral $\int_0^{\ln(3)} \sinh(x) \cosh(x) dx$, and fully simplify your answer.
 - (c) Determine the value of the limit $\lim_{x\to 0^+} x^2 \ln(x^2)$.
- 3. (16 points) Find the area of the largest rectangle which is symmetric around the *y*-axis, bounded below by the *x*-axis, and which has two corners touching the graph of $f(x) = \frac{1}{1+x^2}$. Fully justify your answer by using an appropriate test.



- 4. (18 points) A bug flying in a straight line starts decelerating at time t = 0 at a constant rate of 1 ft/s² for 5 seconds. Answer the following questions about the bug over the time interval $0 \le t \le 5$.
 - (a) Find the bug's velocity as a function of time, given that its velocity at t = 0 is 2 ft/s.
 - (b) What is the bug's displacement over the time interval $0 \le t \le 5$?
 - (c) The bug changes direction at least once during the 5 seconds. What is the total distance the bug travels over the time interval $0 \le t \le 5$?

5. (12 points) For what value of a is the following function continuous?

$$f(x) = \begin{cases} 2x^2 - x + a, & x \le 0\\ \\ \frac{x}{2\sin(x)}, & x > 0 \end{cases}$$

Justify your answer with appropriate computations.

6. (18 points) Consider the function

$$g(x) = \arctan(x) + \frac{1}{x^2 - 4}$$

- (a) Find the domain of the function, and give your answer in interval notation.
- (b) Find all horizontal asymptotes of g(x), and justify your answer with limits.
- 7. (16 points) The half-life of the chemical element cobalt-56 is approximately 77 days. Suppose we have a 10 milligram sample of cobalt-56.
 - (a) Find a formula for the mass of cobalt-56 remaining after t days.
 - (b) How long will it take for only 1 milligram of cobalt-56 to remain in the sample? It is OK for your answer to have a logarithm in it.
- 8. (16 points) For each of the following questions, give a short justification for your answer.

(a) If
$$f(x)$$
 is an odd function and $\int_{-3}^{0} f(x) dx = \pi + 1$, find $\int_{-3}^{3} f(x) dx$.

- (b) Find the absolute minimum of the function $f(x) = x \cdot 2^x$, if it exists.
- (c) Evaluate the limit $\lim_{h \to 0} \frac{\arctan(3x+3h) \arctan(3x)}{h}$.
- (d) Suppose that f(x) is differentiable everywhere, with f(-1) = 1 and f(1) = 3. Is there some value c such that f'(c) = 1?