1. (24 points) The following problems are not related. If a limit does not exist, you must say so. If you use a theorem, clearly state its name and show that its hypotheses are satisfied.
   (Reminder: You may not use L'Hôpital's Rule or "Dominance of Powers" in any solutions on this exam.)

   (a) $\lim_{x\to 0} \dfrac{\sec x}{4x \cot 2x}$

   (b) $\lim_{x\to \infty} \dfrac{\sin^2 x}{x}$

   (c) $\lim_{x\to 1} \dfrac{x-1}{2-\sqrt{5-x^2}}$

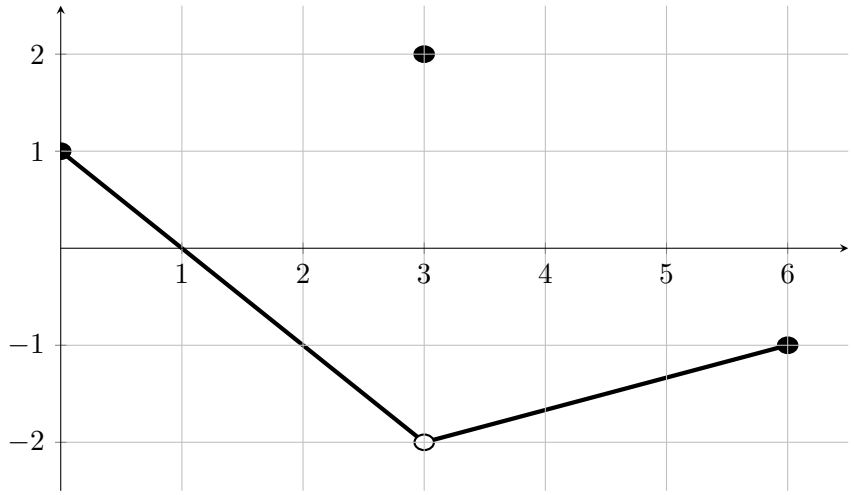
2. (21 points) The following problems are unrelated.

   (a) Given that $\csc\theta = \sqrt{5}$ and $\pi/2 < \theta < \pi$, find the values of $\tan\theta$ and $\cos(2\theta)$.

   (b) Find all values of x in the interval $[0, \pi]$ that satisfy $\tan x \sec x = 4 \sin x$.

   (c) A squirrel is up a tree, and it sees a peanut on the ground some distance away. If the straight-line distance between the peanut and the squirrel is 50 ft, and the angle between the straight-line and the tree is $\pi/6$ radians, how far down the tree and across the ground must the squirrel travel to reach the peanut? *Give your answer with appropriate units.*

3. (15 points) Shown below is a graph of $y = f(x)$, which consists of two line segments with a single removable discontinuity.



   (a) Find a formula for $f(x)$.

   (b) Sketch a graph of $y = |f(x)| + 1$. Label the intercepts, if any.

   (c) Suppose we use the precise definition of a limit to verify the value of $\lim_{x\to 5} f(x)$, and we find that if $4 < x < 6$, then $-\frac{5}{3} < f(x) < -1$. What are the corresponding values of ϵ and δ? (recall the precise definition of a limit: *the limit of $f(x)$ as x approaches a is L if for every number $\epsilon > 0$, there is a corresponding $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.*

4. (20 points) Consider the function $g(x) = \dfrac{2x^2 - 12x + 16}{x^2 - 7x + 12}$.

   (Reminder: You may not use L'Hôpital's Rule or "Dominance of Powers" in any solutions on this exam.)

   (a) Find the domain of $g(x)$. Express your answer in interval notation.

   (b) Find and classify all discontinuities of $g(x)$; justify your answers by calculating the appropriate limits.

   (c) Find the horizontal asymptotes, if any; justify your answers by calculating the appropriate limits.

5. (10 points) Consider the function

$$f(x) = \begin{cases} b\cos(\pi x), & x \leq 1 \\ 3 - \sqrt{2x-2}, & x > 1 \end{cases}$$

   Find the value of b such that $\lim\limits_{x \to 1} f(x)$ exists. Justify your answer by calculating appropriate limits.

6. (10 points) Show that the equation $x - 2 = \sin x \cos x$ has at least one real solution. Indicate the interval where a solution can be found.