1. (30 pts) Evaluate the following expressions.

   (a) $\displaystyle\int \frac{5x^3 - 3x^2 + 2x}{x\sqrt{x}}\, dx$

   (b) $\displaystyle\int \frac{1}{x^3} \sec\left(1/x^2\right) \tan\left(1/x^2\right) dx$

   (c) $\displaystyle\frac{d}{dx} \int_{x^2}^{\cos x} \frac{t}{t^3+4}\, dt$

   Solution:

   (a) $\displaystyle\int \frac{5x^3 - 3x^2 + 2x}{x\sqrt{x}}\, dx = \int \left(5x^{3/2} - 3x^{1/2} + 2x^{-1/2}\right) dx = \boxed{2x^{5/2} - 2x^{3/2} + 4x^{1/2} + C}$

   (b) Let $u = 1/x^2$, $du = -2/x^3\, dx$.

   $$\int \frac{1}{x^3} \sec\left(1/x^2\right) \tan\left(1/x^2\right) dx = \int -\frac{1}{2} \sec u \tan u\, du = -\frac{1}{2} \sec u + C = \boxed{-\frac{1}{2} \sec\left(1/x^2\right) + C}$$

   (c) By FTC-1 and the chain rule:

   $$\frac{d}{dx} \int_{x^2}^{\cos x} \frac{t}{t^3+4}\, dt = \frac{d}{dx} \left(\int_a^{\cos x} \frac{t}{t^3+4}\, dt - \int_a^{x^2} \frac{t}{t^3+4}\, dt \right) = \boxed{\frac{-\cos x \sin x}{\cos^3 x + 4} - \frac{2x^3}{x^6+4}}$$

2. (12 pts) Verona Rupes is a cliff on Uranus's moon Miranda, 20,000 meters tall. The gravitational pull on the moon is $a(t) = -\frac{2}{25}$ meter/sec^2. Suppose a rock is dropped off the top of the cliff.

   Justify your answers to the following questions using calculus techniques. Write your answer in simplest radical form.

   (a) How long would it take for the rock to hit the ground?

   (b) How fast would the rock be going when it hit?

   Solution:

   (a) First find $v(t)$ using the initial value $v(0) = 0$.

   $$v(t) = \int a(t)\, dt = \int -\frac{2}{25}\, dt = -\frac{2}{25} t + C$$

   Given $v(0) = 0$, $C = 0$, so

   $$v(t) = -\frac{2}{25} t.$$

   Next find $s(t)$ using the initial value $s(0) = 20000$.

   $$s(t) = \int v(t)\, dt = \int -\frac{2}{25} t\, dt = -\frac{1}{25} t^2 + D$$

Given $s(0) = 20000$, $D = 20000$, so

$$s(t) = -\frac{1}{25}t^2 + 20000.$$

The rock hits the ground when $s(t) = 0$. Solve for t.

$$\begin{aligned} 0 &= -\frac{1}{25}t^2 + 20000 \\ t^2 &= 25 \cdot 20000 = 5 \cdot 10^5 = 500000 \\ t &= \boxed{100\sqrt{50}} \text{ sec} = \boxed{500\sqrt{2}} \text{ sec} \end{aligned}$$

(b) The velocity when the rock hits is

$$v\left(500\sqrt{2}\right) = -\frac{2}{25} \cdot 500\sqrt{2} = \boxed{-40\sqrt{2}} \text{ m/sec} = \boxed{-8\sqrt{50}} \text{ m/sec.}$$

3. (20 pts) The following problems are not related.

(a) Find the slant asymptote of $y = \dfrac{6x^2 - x + 1}{2x + 3}$.

Solution:

$$\begin{array}{r|rrr} & 3x & -5 & \\ \hline 2x+3 & 6x^2 & -x & +1 \\ & -6x^2 & -9x & \\ \hline & & -10x & +1 \\ & & 10x & +15 \\ \hline & & & 16 \end{array}$$

The slant asymptote is $\boxed{y = 3x - 5}$.

(b) Find the sum $\sum_{i=1}^{100}(2i + 23)$ by applying sigma notation rules. Fully simplify your answer.

Solution:

$$\sum_{i=1}^{100}(2i + 23) = 2\sum_{i=1}^{100} i + \sum_{i=1}^{100} 23 = 2 \cdot \frac{100 \cdot 101}{2} + 2300 = 10100 + 2300 = \boxed{12400}$$

(c) Suppose Newton's Method is applied to the continuous function $h(x)$. The equations for the tangent lines to $h(x)$ at five points on the curve are given below. Starting with an initial approximation of $x_1 = 0$, find the next two approximations x_2 and x_3. No justification is necessary for this problem.

Point	Tangent Line
$(-2, 0)$	$y = 10x + 20$
$(-1, 5)$	$y = x + 6$
$(0, 4)$	$y = -2x + 4$
$(1, 3)$	$y = x + 2$
$(2, 8)$	$y = 10x - 12$

Solution:

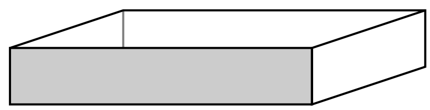
At $x_1 = 0$, the tangent line $y = -2x + 4$ has an x-intercept of 2, so $\boxed{x_2 = 2}$.

Alternatively apply the formula $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = 0 - \dfrac{4}{-2} = 2$.

At $x_2 = 2$, the tangent line $y = 10x - 12$ has an x-intercept of $\dfrac{12}{10} = \dfrac{6}{5}$, so $\boxed{x_3 = \dfrac{6}{5}}$.

Alternatively apply the formula $x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = 2 - \dfrac{8}{10} = \dfrac{6}{5}$.

4. (12 pts) An open-top rectangular box will have a height of 1 foot and a base area of 72 square feet. The front side of the box will cost 3 times as much per square foot as the base and the other three sides. What base dimensions will minimize the cost of materials?



Solution: Let the base have dimensions x by y feet, with the front side of the box measuring x by 1 feet. Then the area of the base is $xy = 72 \implies y = \frac{72}{x}$, the area of the front/back side is x, and the area of the left/right side is y. The cost of materials is

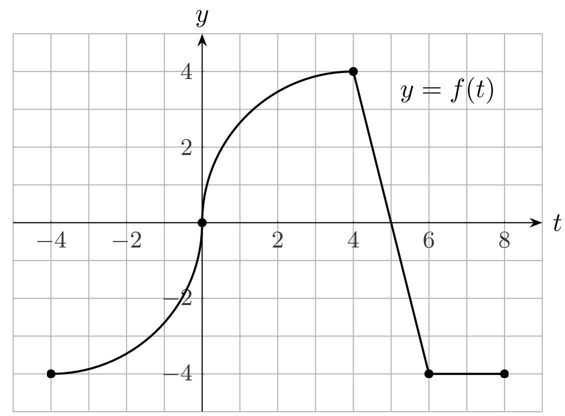
$$\begin{aligned} C &= 3(x) + 1(x + 2y + xy) = 4x + 2y + xy \\ &= 4x + 2 \cdot \frac{72}{x} + 72 \\ &= 4x + \frac{144}{x} + 72. \end{aligned}$$

Solve $C' = 0$ to find the critical number.

$$\begin{aligned} C' &= 4 - \frac{144}{x^2} + 0 = 0 \\ x^2 &= \frac{144}{4} = 36 \\ x &= 6 \end{aligned}$$

Because $C'' = \frac{288}{x^3}$ and $C''(6) > 0$, there is a minimum value at $x = 6$ and $y = \frac{72}{6} = 12$. Therefore the optimal base dimensions are $\boxed{\text{6 ft}}$ for the front/back sides and $\boxed{\text{12 ft}}$ for the left/right sides.

5. (26 pts) Consider the continuous function $f(t)$ defined on $[-4, 8]$, shown below, consisting of two quarter-circles and two line segments.



(a) Find R_3, the right-endpoint approximation of $\int_{-4}^{8} f(t)\,dt$ on $n = 3$ subintervals.

(b) Find the average value of $f(t)$ on $[-4, 8]$.

(c) Let $g(x) = \int_{-4}^{x} f(t)\,dt$, $-4 \le x \le 8$. No justification is necessary for the following questions. Write NONE if appropriate.

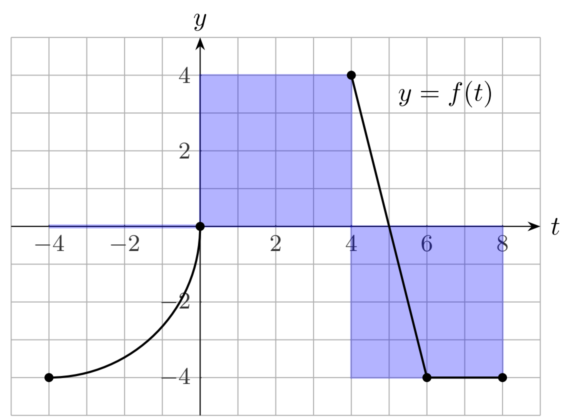
i. At what value(s) of x does $g(x) = 0$?

ii. On what interval(s) is g decreasing?

iii. On what interval(s) is g concave down?

Solution:

(a) $R_3 = \Delta x\,(f(0) + f(4) + f(8)) = 4(0 + 4 - 4) = \boxed{0}$.



(b) The area bounded by a quarter-circle of radius 4 is $\frac{1}{4}(16\pi) = 4\pi$. The net area of the 2 quarter-circular regions plus 2 triangles plus a rectangle is
$\int_{-4}^{8} f(t)\,dt = -4\pi + 4\pi + 2 - 2 - 8 = -8.$

The average value is $f_{\text{ave}} = \dfrac{1}{8-(-4)} \displaystyle\int_{-4}^{8} f(t)\,dt = \dfrac{1}{12}(-8) = \boxed{-\dfrac{2}{3}}$.

Alternate solution:
Using symmetry, $\displaystyle\int_{-4}^{4} f(t)\,dt = 0$ and $\displaystyle\int_{4}^{6} f(x)\,dx = 0$, so $\displaystyle\int_{-4}^{8} f(t)\,dt = \int_{6}^{8} f(t)\,dt = 2(-4) = -8$.

(c) i. $x = \boxed{-4, 4, 6}$ by symmetry

ii. $\boxed{(-4,0),(5,8)}$ where $g'(x) = f(x) < 0$

iii. $\boxed{(4,6)}$ where $g''(x) = f'(x) < 0$