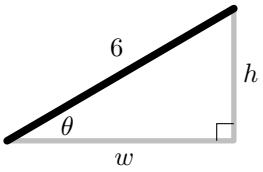
1. (22 pts)

   (a) A 6-meter ramp forms an angle of θ with the ground, $0 < \theta < \frac{\pi}{2}$. The ramp has a horizontal width of w meters and a vertical height of h meters, as shown in the diagram.



   i. If $\csc\theta = 6$, what is the value of w?

   ii. For what value(s) of θ will $w = \frac{h}{\sqrt{3}}$?

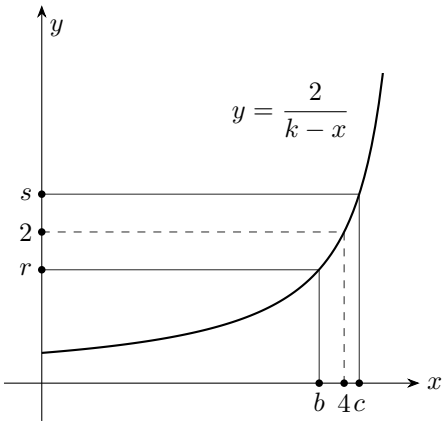
   (b) Find all values of θ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfy $\sec\theta\tan\theta = 2\sin\theta$.

2. (18 pts) A cyclist covered 12 miles in 30 minutes, traveling at a constant rate. Over the next 20 minutes, the cyclist traveled at a slower constant rate and covered 6 miles.

   (a) Write a formula for $d(t)$, the distance in miles traveled by the cyclist as a function of time t in minutes, $0 \le t \le 50$. Simplify your answer.

   (b) Find the cyclist's average speed (in miles per minute) over the interval $t = 20$ to 40 minutes.

3. (12 pts)



   The graph above uses the precise definition of a limit to illustrate $\lim_{x\to 4}\left(\frac{2}{k-x}\right) = 2$ for some constant k with $\epsilon = \frac{1}{2}$. Find the values of k, r, s, b, c, and a corresponding δ.

4. (25 pts) Evaluate the following limits. Fully simplify your answers. (*Reminder:* You may not use L'Hopital's Rule or "dominance of powers" arguments.)

   (a) $\lim_{u \to 13} \dfrac{u - 13}{\sqrt{u} - \sqrt{13}}$

   (b) $\lim_{\theta \to 0} \dfrac{7\theta \cot(3\theta)}{\sec(5\theta)}$

   (c) $\lim_{x \to -3} \dfrac{3 - |x|}{9 - x^2}$

5. (23 pts) The following two problems are not related.

   (a) Let $f(x) = \dfrac{5x + 15}{x^2 - 4x - 21}$. Determine all values of x at which $f(x)$ is discontinuous. For each value, identify the type of discontinuity. Justify using appropriate limits.

   (b) Prove that $P(x) = 3x^3 - 4x^2 - 3x + 1$ has at least **two real roots** between $x = 0$ and $x = 2$.