APPM 1350
Spring 2022

Exam 1

February 3

Instructions:

- Write your name and section number at the top of each page.
- Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified.
- Name any theorem that you use and explain how it is used.
- Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to upload your work.
- When you have completed the exam, email your proctor and request permission to begin scanning. Upload to Gradescope. Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave the room.

Half / Double Angle Formulas

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 1 - 2\sin^2(\theta) \\ 1 + 2\cos^2(\theta) \end{cases}$
- $\tan(2\theta) = \dfrac{2\tan(\theta)}{1 - \tan^2(\theta)}$

- $\sin\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1}{2}\left(1 - \cos(\theta)\right)}$
- $\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1}{2}\left(1 + \cos(\theta)\right)}$
- $\tan\left(\dfrac{\theta}{2}\right) = \begin{cases} \pm\sqrt{\dfrac{1-\cos(\theta)}{1+\cos(\theta)}} \\ \dfrac{\sin(\theta)}{1+\cos(\theta)} \\ \dfrac{1-\cos(\theta)}{\sin(\theta)} \end{cases}$

Angle Sum / Difference Formulas

- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$

- $\tan(\alpha \pm \beta) = \dfrac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$

1. (20 pts)

   (a) Find all solutions to the following equation in $[0, 2\pi]$:

   $$\sqrt{2}\sin(2x) + 1 = 0.$$

   (b) If $\tan(u) = \dfrac{3}{4}$ for u in $(\pi, \dfrac{3\pi}{2})$, find $\sin(u)$ and $\cos(u)$.

2. (20 pts)

   (a) The intensity of a light follows the inverse square law

   $$I(x) = \frac{k}{x^2}$$

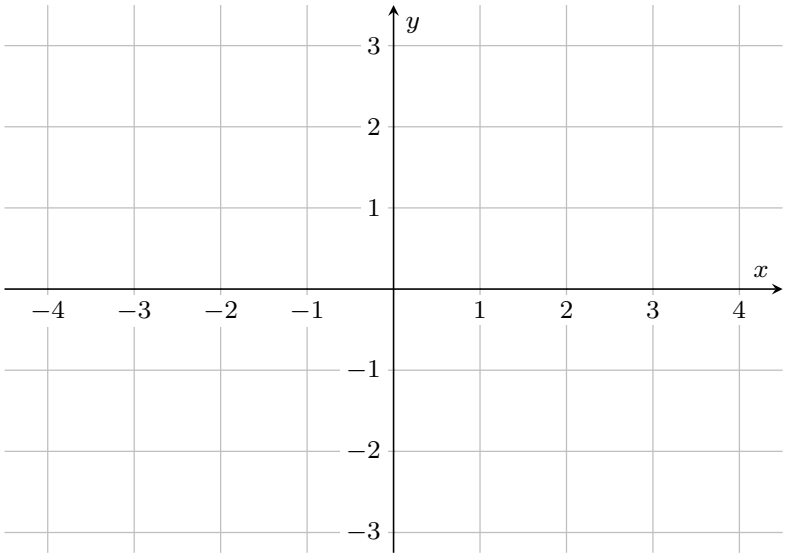
   where x is the distance from the light in meters. If the intensity of the light at 3 meters is 20 lumens find the intensity of the light at 5 meters.

   (b) The function $g(x)$ is obtained from the function $f(x) = \sqrt{x}$ by reflecting across the y-axis, shifting to the right by 2, then shifted down by 1.

   i. Find a formula for the new function $g(x)$ obtained by the transformations of $f(x)$.

   ii. Sketch a graph of f and sketch a graph of g on the same axes.

$f(x)$ and $g(x)$



3. (26 pts) Evaluate the following limits or show that they do not exist.

   (a) $\displaystyle\lim_{x\to -1} \frac{|x+1|}{2x+2}$

   (b) $\displaystyle\lim_{x\to 0} \frac{3x\sec(x) - 4\sin(x)}{2\tan(x)}$

   (c) $\displaystyle\lim_{x\to -3^-} \frac{x^2+2x+5}{2x+6}$

4. (34 pts)

(a) Consider the function $f(x) = \dfrac{2-x}{\sqrt{4x+1}-3}$

i. Find the domain of this function.

ii. Show that there is a removable discontinuity at $x = 2$. Use appropriate limit notation.

(b) Show that the following equation has at least one real solution.

$$\sin(x) = 7\cos(x)$$

(c) Consider $w(x) = \dfrac{1+4x}{x}$ and $v(x) = \dfrac{1}{x-3}$. Find $w \circ v$, simplify as much as possible and give its domain.

The End