1. (24pts) Start this problem on a **new** page. The following parts of this problem are not related.

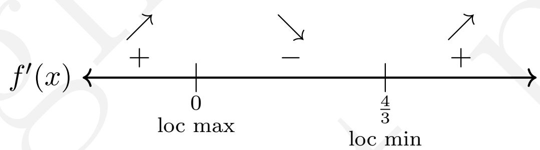
   (a)(12pts) Find and classify the *critical points* of $f(x) = x^3 - 2x^2 + 4$ as being either a *local maximum*, *local minimum*, or *neither* using the **First Derivative Test**. You do not have to find the local extreme y-values. Show all work.

   (b)(12pts) Suppose we want to approximate the x-intercept of $f(x) = 3x^2 - 2$ using Newton's Method. What would the formula for x_{n+1} be? (To get full points for this question you must provide the explicit formula for x_{n+1} in terms of x_n, the generic formula for Newton's Method is not sufficient. You do *not* need to approximate the solution. **Simplify your answer.**)

   Solution: (a)(12pts) Note that

$$f'(x) = 0 \Rightarrow = [x^3 - 2x^2 + 4]' = 0 \Rightarrow 3x^2 - 4x = 0 \Rightarrow x(3x-4) = 0 \Rightarrow x = 0, \frac{4}{3}$$

   thus the only critical points are $x = 0, \frac{4}{3}$ since $f(x)$ is a polynomial. Now applying the First Derivative Test yields:



   (for example, note that $f'(-1) = 7 > 0$, $f'(1) = -1 < 0$ and $f'(2) = 4 > 0$).

   Thus, we have a $\boxed{\text{local maximum at } x = 0}$ and a $\boxed{\text{local minimum at } x = \frac{4}{3}}$ (by the First Derivative Test).

   (b)(12pts) Note that (by Newton's Method) we have

$$f'(x) = [3x^2 - 2]' = 6x \quad \Rightarrow \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{3x_n^2 - 2}{6x_n} = \frac{6x_n^2 - (3x_n^2 - 2)}{6x_n} = \frac{3x_n^2 + 2}{6x_n}$$

$$\Rightarrow \quad \boxed{x_{n+1} = \frac{3x_n^2 + 2}{6x_n}, \; n = 1, 2, \ldots}$$

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2. (28pts) Start this problem on a **new** page. The following parts of this problem are not related.

   (a)(12pts) Find the most general antiderivative of $f(x) = \dfrac{5}{\sqrt{x}} + \sec^2(x)$. Justify your answer.

   (b)(12pts) Approximate the area under the curve $x^2 + 2x + 4$ from $x = 0$ to $x = 6$ with a *Riemann sum* using $n = 3$ subintervals of equal width and left endpoints (that is, find the approximation L_3).

   (c)(4pts) *(Multiple Choice)* Using right endpoints (R_n) and subintervals of equal width, which limit below is equal to $\displaystyle\int_1^3 \frac{x}{x^2+4}\,dx$? (**No justification necessary** - *Choose only one answer, copy down the entire* *answer.)*

(A) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2i/n}{(2i/n)^2+4}\cdot\frac{2}{n}$ (B) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1+2i/n}{(1+2i/n)^2+4}\cdot\frac{2}{n}$ (C) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1+2(i-1)/n}{(1+2(i-1)/n)^2+4}\cdot\frac{2i}{n}$ (D) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2i/n}{(2i/n)^2+4}$

Solution:

(a)(12pts) Note that

$$\begin{aligned}\int\left[\frac{5}{\sqrt{x}}+\sec^2(x)\right]dx &= \int 5x^{-1/2}\,dx+\int\sec^2(x)\,dx\\ &= 5\cdot\frac{x^{-1/2+1}}{-\frac{1}{2}+1}+\tan(x)+C = 5\cdot\frac{x^{1/2}}{\frac{1}{2}}+\tan(x)+C = 10\sqrt{x}+\tan(x)+C\end{aligned}$$

Thus, we get $\boxed{F(x)=10\sqrt{x}+\tan(x)+C.}$

(b)(12pts) Note that $\Delta x=\frac{6-0}{3}=2$ implies that $x_0=0$, $x_1=2$, $x_2=4$ and $x_3=6$, thus, using left endpoints yields the approximation

$$\int_0^6[x^2+2x+4]\,dx\approx L_3=\sum_{i=1}^{3}f(x_{i-1})\Delta x=f(0)\cdot 2+f(2)\cdot 2+f(4)\cdot 2=2\,[4+12+28]=2\cdot 44=\boxed{88.}$$

(c)(4pts) $\boxed{\text{Choice (B).}}$ Discussion: We know that $a=1$ and $b=3$ from the integral, so $\Delta x=(3-1)/n=2/n$. Likewise, using right endpoints, we find that $x_i=1+2i/n$. Combining this information into the limit definition, we get $\lim_{n\to\infty}\sum_{i=1}^{n}f(x_i)\Delta x=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1+2i/n}{(1+2i/n)^2+4}\frac{2}{n}$ which implies choice (B).

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3. (24pts) Start this problem on a **new** page. The following parts of this problem are not related.

(a) (12pts) Evaluate the integral $\int_0^1|x^2-2x|\,dx$. Simplify your answer.

(b) (12pts) Find numbers x and y satisfying the equation $3x+y=12$ such that the product of x and y is as large as possible. Justify your answer by classifying your critical point(s) using the *Second Derivative Test.* **Be sure to answer the question being asked.**

Solution:

(a)(12pts) Note that $x^2-2x=x(x-2)\le 0$ for $x\in[0,1]$ thus

$$\int_0^1|x^2-2x|\,dx=\int_0^1-(x^2-2x)\,dx=x^2-\frac{x^3}{3}\Big|_0^1=1-\frac{1}{3}=\boxed{\frac{2}{3}.}$$

(b)(12pts) We wish to *maximize* the function $A(x,y)=xy$ subject to $3x+y=12\Rightarrow y=12-3x$ thus we wish to maximize

$$A=xy=x(12-3x)=12x-3x^2\ \Rightarrow\ \frac{dA}{dx}=12-6x=0\ \Rightarrow\ x=2$$

and note that since this is the only critical point of $A(x)$ and $\frac{d^2A}{dx}=-6<0$, we have a local maximum at $x=2$ (by Second Derivative Test) that is also an absolute maximum. Thus the numbers x and y satisfying the equation $3x+y=12$ such that the product of xy is as large as possible are $\boxed{x=2\text{ and }y=12-3(2)=6.}$

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4. (24pts) Start this problem on a **new** page. The following parts of this problem are not related.

(a)(10pts) Evaluate the definite integral $\displaystyle\int_1^4 \frac{1+x^{5/2}}{x^{1/2}}\,dx$. Simplify your answer.

(b)(10pts) If $g(t)$ is *integrable* and $\displaystyle\int_0^1 g(t)\,dt = 5\sqrt{5}-8$, find $\displaystyle\int_1^0 \Big[g(t)-10\Big]\,dt$. Show all work.

(c)(4pts) Which graph below best matches the graph of the function $f(x)$ given that $f'(0) = -3$ and $f''(x) = 6x$? **Choose only <u>one</u> answer.** *No justification necessary, clearly indicate your answer otherwise points will be deducted.*

Solution: (a)(10pts) Note that

$$\begin{aligned}\int_1^4 \frac{1+x^{5/2}}{x^{1/2}}\,dx &= \int_1^4 \left(\frac{1}{x^{1/2}}+\frac{x^{5/2}}{x^{1/2}}\right)dx\\ &= \int_1^4 \left(x^{-1/2}+x^2\right)\,dx\\ &= 2x^{1/2}+\frac{x^3}{3}\Big|_1^4 = \left(2\cdot 2+\frac{64}{3}\right)-\left(2+\frac{1}{3}\right) = 2+\frac{63}{3} = \frac{69}{3} = \boxed{23.}\end{aligned}$$

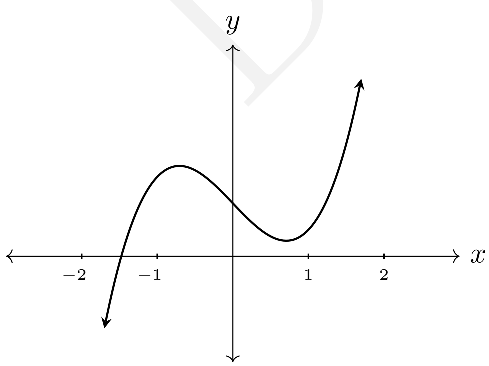
(b)(10pts) Note that,

$$\begin{aligned}\int_1^0 \Big[g(t)-10\Big]\,dt &= -\int_0^1 \Big[g(t)-10\Big]\,dt\\ &= -\int_0^1 g(t)\,dt + \int_0^1 10\,dt\\ &= -\left(5\sqrt{5}-8\right)+10(1-0) = -5\sqrt{5}+8+10 = \boxed{18-5\sqrt{5}.}\end{aligned}$$

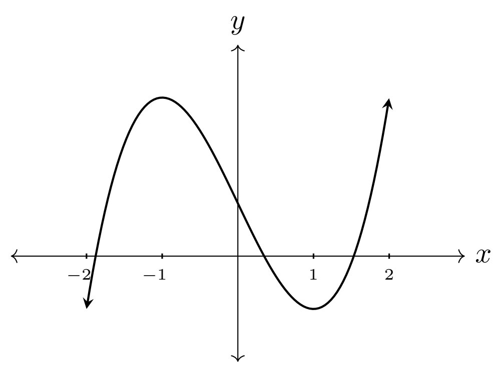
(c)(4pts) $\boxed{\text{Choice (D).}}$ <u>Discussion:</u> First note that if $f'(0) = -3$ then choice (B) is not possible and, furthermore, the concavity information from the fact that $f''(x) = 6x$ implies that we can eliminate choice (A) (and choice (B) once again) and finally note that

$$f''(x) = 6x \;\Rightarrow\; f'(x) = 3x^2 + C \;\Rightarrow\; -3 = f'(0) = C \;\Rightarrow\; f'(x) = 3x^2 - 3$$

so we have $f'(x) = 0 \iff x = \pm 1$ which allows us to eliminate choice (C) $\Rightarrow$ choice (D).



Graph (C) ✗



Graph (D) ✓