INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your full name**, (2) **1350/Exam 2**, (3) **lecture number/instructor name** and (4) **SPRING 2021** on the first page of your test. Do all problems. **Start each problem on a new page.** $\boxed{\text{Box}}$ **your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work.** You must start uploading your test to Gradescope by 9:01pm.

Write the **honor code statement** given in the box below on the **first page** of your test and **sign** and **date it**.

On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this work. Signature:_____ Date:_____

1. (28pts) Start this problem on a **new** page. The following parts of this problem are not related.

    (a)(12pts) Find the equation of the *tangent line* to $f(x) = x^3 + 2\sqrt{x}$ at $x = 1$. Simplify your answer.

    (b)(12pts) Given $F(x) = \dfrac{2x^3}{4x+1}$, find the derivative using the *Quotient Rule*. Simplify the numerator of your answer.

    (c)(4pts) *(Multiple Choice)* Use *differentials* to identify the maximum error of the volume of a cube whose sides are $10\,\text{ft}$ in length, if the maximum error of the length is $0.5\,\text{ft}$. (**No justification necessary** - *Choose only one* answer from below, copy down the entire answer.)

    (A) $125\,\text{ft}^3$ (B) $300\,\text{ft}^3$ (C) $50\,\text{ft}^3$ (D) $150\,\text{ft}^3$ (E) None of these

2. (24pts) Start this problem on a **new** page. The following parts of this problem are not related.

    (a)(12pts) For what values of a and b will the function $f(x) = \begin{cases} ax^2 - 3, & \text{if } x \leq -1 \\ a + bx, & \text{if } x > -1 \end{cases}$ be *differentiable* at $x = -1$? Justify your answer.

    (b)(12pts) Find the derivative of $f(x) = 2\sin\left(3x + \tan(x)\right)$ at $x = 0$. Show all work.

PROBLEMS #3 & #4 ON THE NEXT PAGE

3. (24pts) Start this problem on a **new** page. The following parts of this problem are not related.

(a)(10pts) Find the *absolute* minimum and maximum values of $f(x) = \sqrt{x}(x-6) = x^{3/2} - 6x^{1/2}$ on the interval $[0, 4]$. Give your answer in the form (x, y). Show all work, justify your answers and clearly label your answers.

(b)(10pts) Find dy/dx by *implicit differentiation* given that $x^3 - 2y = xy^2$. Simplify your answer.
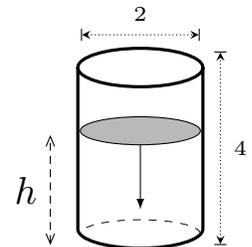
(c)(4pts) *(Multiple Choice)* If $u(x)$ is a differentiable function of x such that $u(2) = -5 = u'(2)$ and if $f(u) = u^2$ then $\left. \dfrac{df}{dx} \right|_{x=2}$ is equal to which choice below? (**No justification necessary** - *Choose only one answer, copy down the entire answer.*)

    (A) 25   (B) 50   (C) 125   (D) -125 (E) None of these

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4. (24pts) Start this problem on a **new** page. The following parts of this problem are not related.

(12pts) (a)(*i*)(6pts) Explain why the function $f(x) = x^3 - 3x + 2$ satisfies the hypothesis of the Mean Value Theorem on the interval $[-2, 2]$. (a)(*ii*)(6pts) Find all numbers $c \in (-2, 2)$ that satisfy the conclusion of the Mean Value Theorem. **Explanations should be in complete sentences**, show all work.

(b)(12pts) A water heater that has the shape of a cylindrical tank with a diameter of $2\,\mathrm{m}$ and a height of $4\,\mathrm{m}$ is being drained (see the diagram on the right). How fast is water draining out of the tank (in $\mathrm{m}^3/\mathrm{min}$) if the water level is dropping at $50\,\mathrm{cm}/\mathrm{min}$? **Be sure to answer the question being asked and give your final answer in the form of a complete sentence.**

— **END** —
(STAY SAFE.)