THIS EXAM MUST BE COMPLETED ON YOUR OWN PAPER, SCANNED AND UPLOADED TO CANVAS TO THE FINAL EXAM ASSIGNMENT BY 1:10 PM MDT.

- Show **all** your work, simplifying and putting a box around your final answer.
- No calculators, cell phones, or other electronic devices are permitted (except for those used to attend the Zoom meeting).

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1. (30 pts) Evaluate the following

   (a) $\displaystyle\int_0^1 \frac{3}{9+x^2}\,dx$

   (b) $\displaystyle\int x\cosh(x^2)\,dx$

   (c) $\displaystyle\int_0^{\ln(\sqrt{3})} \frac{e^{2x}}{\sqrt{3+e^{2x}}}\,dx$

2. (25 pts) Compute the following limits

   (a) $\displaystyle\lim_{x\to-\infty} \frac{x^2}{e^{1-x}}$

   (b) $\displaystyle\lim_{x\to 0} \arctan(x)\cos\left(\frac{2}{x}\right)$ (hint: use the squeeze theorem)

   (c) $\displaystyle\lim_{\underbrace{x}_{\text{typo}}\to-1^-} \frac{t+1}{|t+1|}$

3. (25 pts) The following problems are unrelated

   (a) Find y' for $y=\sqrt{x}^{\sqrt{x}}e^{x^2}$.

   (b) Find the tangent line to $f(x)=\ln(x)\log_2(x)$ at $x=2$.

   (c) Approximate the value of $\displaystyle\int_0^1 3\sin^{-1}(x)dx$ using R_2, the right-endpoint approximation with two equal subintervals. Simplify your answer.

4. (15 pts) In this problem we consider the function $f(x)=3xe^{2x-10}$ at $x=5$.

   (a) Find the linearization of $f(x)$ at $x=5$.

   (b) Find the interval(s) on which the function is increasing.

5. (20 pts) A colony of bacteria is growing exponentially. If there are 200 bacteria after 2 hours and 800 bacteria after 5 hours. Find the growth rate k.

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6. (15 pts) A boat leaves a dock traveling due south at 20 km/hour. At the same time, a boat 20 km due west from the dock begins traveling east at 10 km/hour. At what time is the distance between the two boats a minimum? (hint: write the coordinate of each boat as a function of time)

7. (20 pts) The following problems consider a function $f(x)$ that is continuous everywhere with the following properties

- $f'(x) > 0$ on $(-\infty, 1)$, $f'(x) < 0$ on $(1, \infty)$ and $f'(1) = 0$.
- $f''(x) > 0$ on $\underbrace{(\infty, -2)}_{\text{typo}}$ and $(2, \infty)$, $f''(x) < 0$ on $(-2, 2)$.
- $\lim_{x \to -\infty} f(x) = -\infty$.
- $\lim_{x \to \infty} f(x) = 1$.
- $f(-3) = -2$
- $f(1) = 2$

(a) Sketch a single curve with all of the above properties.
(b) Can you guarantee that $f(x)$ has a local maximum? You must fully justify your answer, referencing your graph from part (a) is not sufficient.
(c) Can you guarantee that there is a value of c such that $f(c) = 0$? Fully justify your answer.

Formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$