1. (20 pts) Find the requested derivative for each of the following.

(a) Find $y'$ for $y = \sqrt[3]{x} \cos(x)$.

(b) Find $\frac{dy}{dx}$ for $(x + y)^3 = x^3 + y^3$.

(c) Find $\frac{d^2y}{dt^2}$ for $y = \sec(t)$.

(d) Find $f'(1)$ for $f(x) = \frac{x^2 - 4}{x - 3}$.

2. (16 pts) The following problems are unrelated

(a) Use the definition of a derivative to find the derivative of $f(t) = 10 - t^2$.

(b) Find the linearization of $\tan (x - \pi/4)$ at $a = \pi/2$ and use it to approximate the value of $\tan(\pi/5)$.

3. (24 pts) The position of a particle along a straight line is described by the function

$$s(t) = \frac{t}{4} - \frac{1}{2} \sin(t),$$

on the interval $[0, 2\pi]$.

(a) What is the velocity of the object at any time $t$?

(b) What is the acceleration of the object at any time $t$?

(c) Find all critical values of $s(t)$ on the interval $(0, 2\pi)$.

(d) Use the second derivative test to classify the local extrema of $s(t)$ on the interval $(0, 2\pi)$.

(e) Find the absolute maximum and minimum values of the velocity on the interval $[0, 2\pi]$.

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4. (20 pts) The following problems are unrelated.

(a) (i) State Rolle’s theorem.
(ii) Verify that \( f(x) = x^2 - 2x - 8 \) satisfies the assumptions of Rolle’s theorem on the interval \([-1, 3]\). Then find all values of \( c \) whose existence is guaranteed by Rolle’s theorem on the interval.

(b) Below is a sketch of a function \( g(x) \) on the interval \([a, f]\). Use the values of the \( x \)-coordinates specified to identify the following. No justification is necessary.

(i) Critical numbers of \( g(x) \).
(ii) Locations of the absolute extrema of \( g(x) \).
(iii) Intervals where \( g'(x) \geq 0 \).
(iv) Intervals where \( g''(x) > 0 \).

5. (20 pts) A thin sheet of ice is in the form of a circle and maintains this shape as it melts. If the ice is melting in such a way that the area of the sheet is decreasing at a rate of 0.5 m\(^2\)/sec, at what rate is the radius decreasing when the area of the sheet is 12 m\(^2\)?