APPM 1350 **Exam 1** **Spring 2020**

On the front of your Bluebook write: (1) your name, (2) your instructor's name, (3) your lecture section number and (4) a grading table for ***five*** problems.

- Text books, class notes, cell phones and calculators are NOT permitted.
- **Show all work and simplify your answers!** Name any theorem that you use. Limit problems should not be evaluated using LHopitals Rule. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.

---

1. (20 pts) The following problems are unrelated. A list of helpful trigonometric identities is provided on the back of the exam.

(a) Suppose that $\sec(\theta) = 3$, where $\frac{3\pi}{2} \leq \theta \leq 2\pi$. Find the values of the following

(i) $\sin(\theta)$ (ii) $\cos(\theta)$ (iii) $\cos(2\theta)$

(b) Find all values of x in the interval $[0, \pi]$ that solve $2\cos^2(x) = 2 - \cos(2x)$.

2. (20 pts) Evaluate the following limits and simplify your answers.

(a) $\lim_{x \to -3} \dfrac{\sqrt{2x + 22} - 4}{x + 3}$

(b) $\lim_{x \to \infty} \dfrac{20x^4 - 7x^3}{2x + 9x^2 + 5x^4}$

(c) $\lim_{x \to 0} x^4 \sin\left(\dfrac{\pi}{x}\right)$

3. (20 pts) In this problem, let $f(x) = \sqrt{x - 6}$, $g(x) = |2x - 2|$.

(a) Sketch the function $g(x)$ on the interval $[-2, 2]$. Label all intercepts.

(b) Find $(f \circ g)(x)$.

(c) Evaluate the limit: $\lim_{x \to 1} \dfrac{g(x)}{2 - 2x}$.

(d) Suppose we let $h(x) = \begin{cases} f(x) & \text{if } x > 6 \\ g(x) & \text{if } x \leq 6 \end{cases}$. Are there any values of x for which $h(x)$ is not continuous? Justify your answer and specify the type of discontinuity (i.e. jump, removable, infinite) if there are any.

CONTINUED ON BACK

4. (20 pts) In this problem, we will study the function $f(x) = \dfrac{\cos x \sec x \sin x}{3x}$

(a) Compute $f(\pi)$ and $f(\frac{\pi}{4})$.

(b) Classify $f(x)$ as even, odd, or neither. For full credit you must show your work.

(c) What is $\lim_{x \to 0} f(x)$?

(d) Is $f(x)$ continuous at $x = 0$? If the function is **not** continuous, classify the type of discontinuity.

5. (20 pts) The following problems are unrelated.

(a) Identify all vertical and horizontal asymptotes of the function $f(x) = \dfrac{x+7}{x^2-4}$. Show all your work.

(b) What is the domain of the function $g(x) = \dfrac{1}{\sqrt{x^2(x-1)}}$? Express your answer using interval notation.

(c) Use the Intermediate Value Theorem to show that there is a solution to $x = x^3 + 5$.

Some helpful trigonometric identities

$\sin(2x) = 2\sin(x)\cos(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$

$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) + \sin(x)\sin(y)$