INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your full name**, (2) **1350/Final**, (3) **lecture number/instructor name** and (4) **SPRING 2019** on the front of your bluebook. Make a **grading table** for 5 problems and a total. Do all problems. **Start each problem on a new page.** **Box** **your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work.**

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1. (34pts) The following problems are not related.

   (a)(17pts) Use *logarithmic differentiation* to find $\frac{dy}{dx}$ if $y = \frac{x^x}{\cosh(x)}$.

   (b)(17pts)(i)(8pts) Write down the piecewise definition of the function $f(x) = |e^x - 2|$.

   (ii)(9pts) Evaluate the integral $\int_0^{\ln(4)} |e^x - 2|\, dx$. Simplify your answer.

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2. (35pts) The following problems are not related.

   (a)(16pts) In this problem, you will find two positive integers x and y such that the sum of the first number and four times the second number is 1000 and the product is as large as possible. Answer the following questions: (i)(4pts) Is this is a *minimization* or *maximization* problem? Write down a function in terms of the two variables x and y that you would minimize (or maximize). (ii)(4pts) Write down an equation that relates the variables x and y. (iii)(8pts) Now using optimization find the value of x and y that satisfy this problem. Justify your answer by classifying your critical point(s) using either the 1st or 2nd Derivative Test.

   (b)(16pts) Evaluate the integral, show all work: $\int_e^{e^3} \frac{1}{x\ln(x)}\, dx$.

   (c)(3pts) **Fill in the blank, no justification necessary.** If $f(x) = x^4 + x + 3$ for $0 \le x \le 2$ then the point c in $(0, 2)$ whose existence is guaranteed by the Mean Value Theorem is $c =$ *write answer in bluebook* .

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3. (34pts) The following problems are not related.

   (a)(17pts) Evaluate the limit: $\lim_{x\to 0^+} x^{\sqrt{x}}$. Show all work.

   (b)(17pts) If $f(x) = \int_2^x \frac{1}{\sqrt{1+t^4}}\, dt$, find $(f^{-1})'(0)$. Show all work.

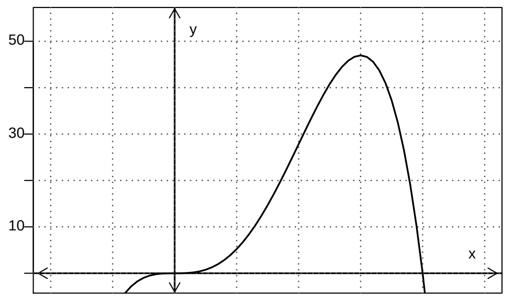
PROBLEMS #4 & #5 ON THE OTHER SIDE

4. (35pts) The following problems are not related.

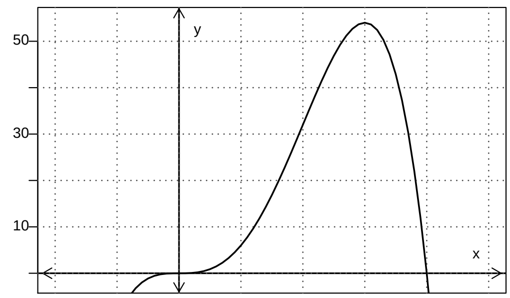
(a)(16pts) Is the function $f(x) = \begin{cases} \dfrac{\sin(-x)}{6x}, & \text{if } x < 0 \\ \dfrac{x^2+x+2}{6x^2-12}, & \text{if } x \geq 0 \end{cases}$ *continuous* at $x = 0$? Justify your answer with limits.

(b)(16pts) Evaluate $\displaystyle\int \frac{1+2x}{1+x^2}\,dx$. (*Hint:* Recall that $\int [f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$.)

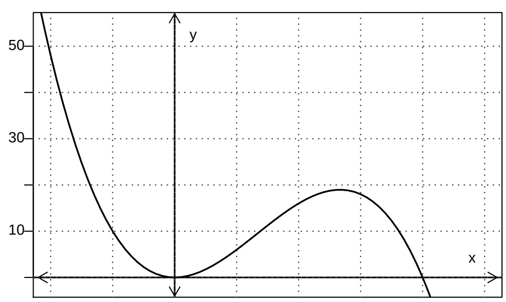
(c)(3pts) Which graph below best matches the graph of the function $f(x) = 8x^3 - 2x^4$? **Choose only one answer.** *No justification necessary, clearly indicate your answer otherwise points will be deducted.*



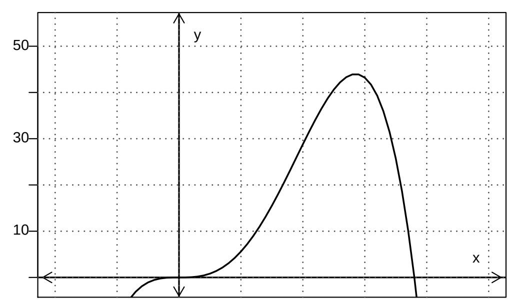
Graph A



Graph B



Graph C



Graph D

5. (12pts) Answer either **ALWAYS TRUE** or **FALSE**. You do NOT need to justify your answer. (*Don't just write down "A.T." or "F", completely write out the words "ALWAYS TRUE" or "FALSE" depending on your answer.*)

(a)(3pts) The linearization of the function $f(x) = \dfrac{e^{2x}}{x}$ at $a = 1$ is $y = e^2 x$.

(b)(3pts) It can be shown that $\displaystyle\lim_{h\to 0} \frac{\sin^{-1}(2x+2h)}{h} - \frac{\sin^{-1}(2x)}{h} = \frac{2}{\sqrt{1-4x^2}}$.

(c)(3pts) The function $f(x) = xe^x$ has a local minimum at $x = -1$.

(d)(3pts) If $f(x)$ is an even function and $\displaystyle\int_0^2 f(x)dx = \pi - 3$ then $\displaystyle\int_{-2}^0 f(x)dx = -\pi + 3$.

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