APPM 1350
Exam 3
Fall 2023

Name	
Instructor	**Lecture Section**

This exam is worth 100 points and has **6 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

Show all work and *simplify* your answers. Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

End of Exam Check List

1. If you finish the exam before 7:45 PM:
   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.
2. If you finish the exam after 7:45 PM:
   - Please wait in your seat until 8:00 PM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.

Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad \cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

1. (24 pts) The following problems are unrelated. For each, be sure to fully simplify your answers.

   (a) Evaluate $\displaystyle\int_0^2 \frac{x+1}{\sqrt{x^2+2x+4}}\,dx$.

   (b) Evaluate $\displaystyle\frac{d}{dx}\left(\int_x^{x^2} \sqrt{t}\cos(t)\,dt\right)$ where $x > 0$.

   (c) Suppose $\displaystyle\int_1^{-3} (2x + h(x))\,dx = 10$ and $\displaystyle\int_0^1 5h(x)\,dx = -8$. Find the average value of $h(x)$ over $[-3, 0]$.

2. (20 pts) The following problems are unrelated. For each, be sure to fully simplify your answers.

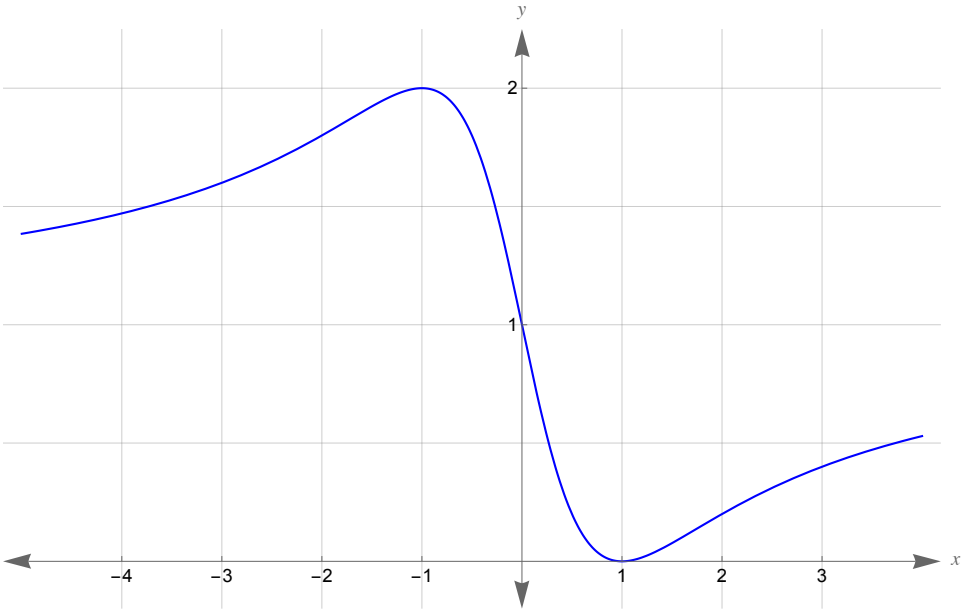
   (a) An object moves along an axis starting at time $t = 0$. It moves with acceleration $\sin(t)$ meters per second squared with an initial velocity of 3 meters per second and an initial position of 2 meters. Find the position of this object at $t = \frac{3\pi}{2}$ seconds. (Provide an exact answer in terms of π. Do not round or approximate your final answer.)

   (b) Consider the integral $\int_0^2 x^3\,dx$. In this problem, you will evaluate this integral in two different ways.

      i. Evaluate this integral using the Fundamental Theorem of Calculus Part 2 (also known as the Evaluation Theorem).

      ii. Separately, write the integral as the limit of a right-hand Riemann sum using n rectangles of equal width. Evaluate this sum and limit without using your work from (i).

3. (10 pts) The following is a graph of $y = f(x)$. Be sure to scan and submit this graph as part of your answer because you will draw on it in addition to answering a few questions (below).



(a) What is the x-value of the only zero of f? (Note: The only zero of f is shown in the above graph.)

(b) With $x_1 = -2$, after one iteration of Newton's Method will x_2 be closer to or farther away from the zero in part (a)?

i. Answer '**closer**' or '**farther**.'

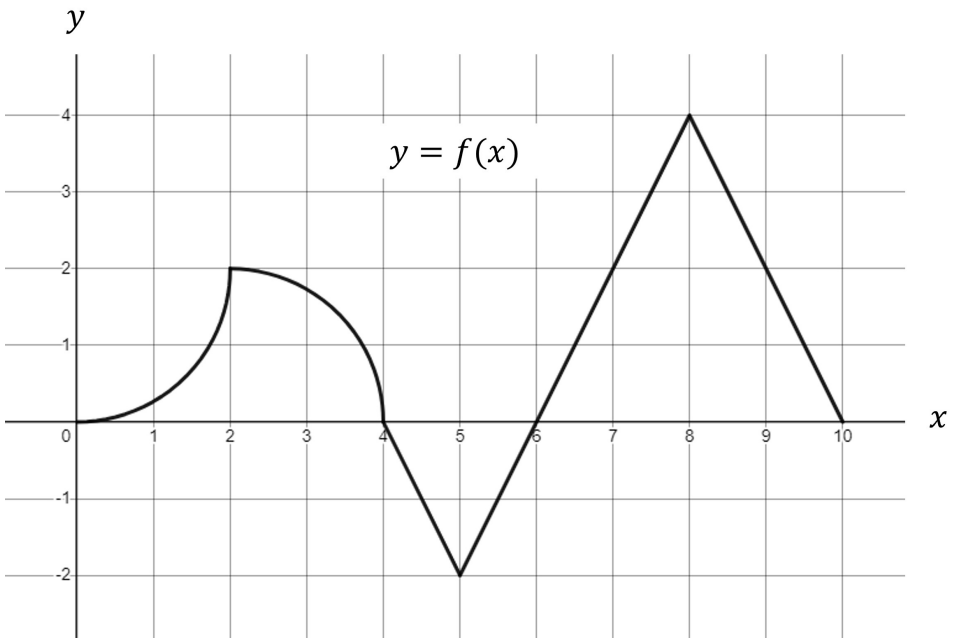
ii. On the provided graph draw the relevant line supporting your answer from (i). *Label this line as (b).*

(c) Give an x-value x_1 for which Newton's method definitely will **not** converge.

(d) With $x_1 = 3$, draw a line on the provided graph illustrating one step of Newton's Method. *Label this line as (d).*

(e) What will $\lim_{n\to\infty} x_n$ equal when $x_1 = 3$? (That is, with an initial guess of $x_1 = 3$ and repeated application of Newton's Method, to what value will the x_n's approach?)

4. (22 pts) Consider the function $f(x)$ defined over $[0, 10]$ that is graphed below. The graph consists of three line segments and two quarter circles.



(a) Approximate $\int_0^{10} f(x)dx$ by using five rectangles of equal width with the righthand endpoint rule.

(b) Determine the exact value of $\int_0^{10} f(x)dx$.

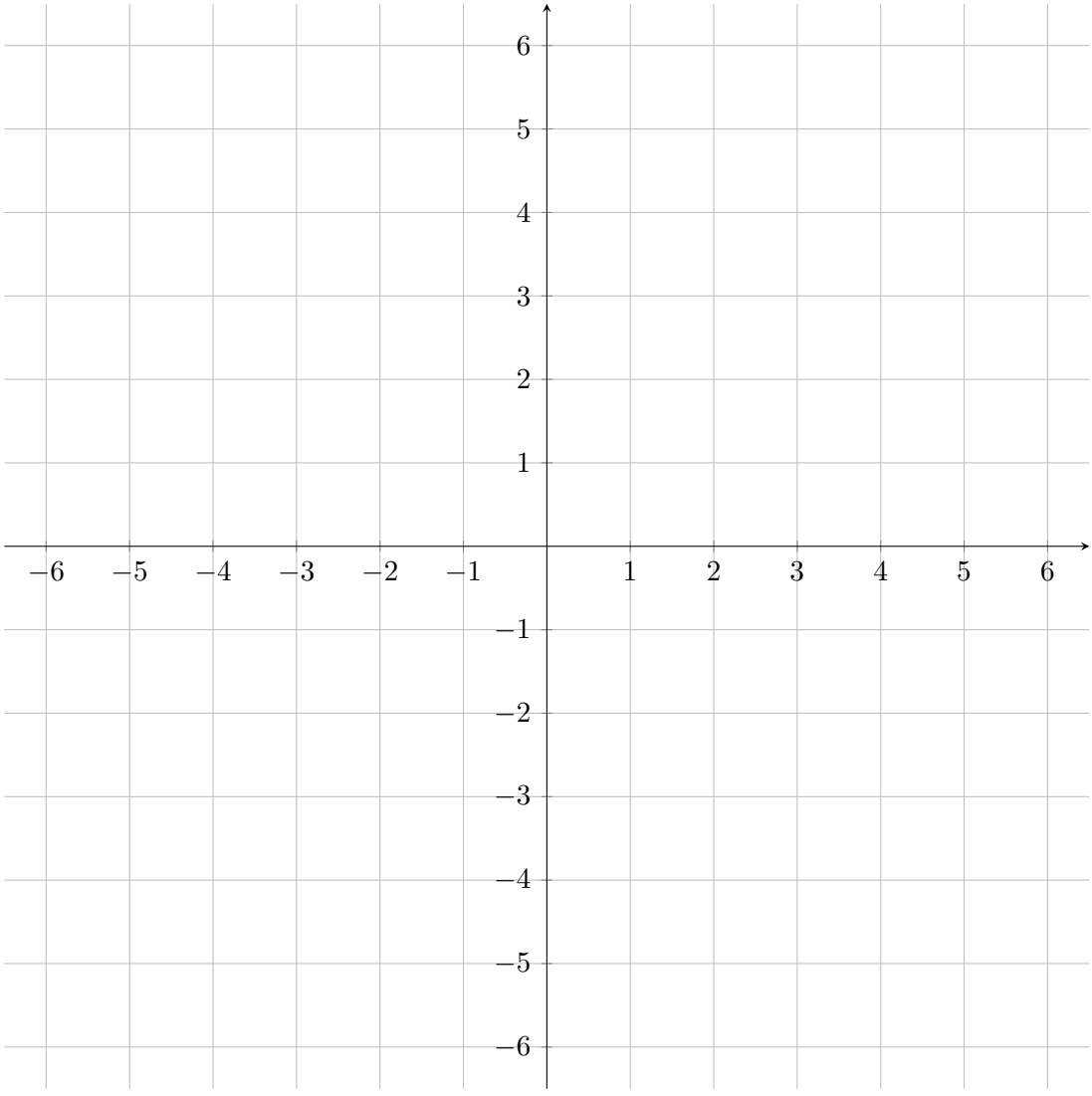
(c) Let $g(x) = \int_0^x f(t)dt$.

i. Identify where the function $g(x)$ is concave up. Express your answer using interval notation.

ii. Given that $g(6) = 2$, find an equation of the line that is tangent to the curve $y = g(x)$ at $x = 7$.

5. (12 pts) Using the grid below, sketch the graph of a **single function**, $y = f(x)$ with each of the following characteristics. (Sketch dashed lines to indicate any asymptotes that are present. The concavity of your graph should be clear.)

$f(-x) = -f(x)$ for all x,	$f'(x) < 0$ for $0 < x < 3$
$f'(3) = 0$,	$f'(x) > 0$ for $x > 3$
$f''(x) < 0$ for $x > 4$,	$f''(x) > 0$ for $0 < x < 4$
$\lim_{x \to -\infty} f(x) = 2$,	f is continuous for all x



6. (12 pts) A box with an open top is to be constructed from a square piece of cardboard, 6 ft by 6 ft, by cutting out identical squares from each of the four corners and bending up the sides.

   (a) Draw a diagram of the flat cardboard sheet with squares cut out of the corners. Make sure you label the length of any parts which are cut out, and also the lengths of the remaining sides.

   (b) What are the dimensions of the square you need to cut out of each corner so that your box has maximum volume? Include the correct units of measurement in your final answer. (Be sure you have justified that you have found the absolute maximum volume.)

   (c) What is the maximum volume of the box? Include the correct units of measurement in your final answer.

END OF TEST

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.