

1. (20 points) Problems (a) and (b) are not related.

(a) Determine all solutions of $\sin^2(x) = \sin(2x) - \sin(x)\cos(x)$ in $[0, 2\pi)$.

(b) If θ is in $[\pi/2, \pi]$ and $\tan(\theta) = -5/8$, evaluate

i. $\sin(\theta)$

ii. $\cos(2\theta)$

2. (24 points) Evaluate the following limits and simplify your answers. If a limit does not exist, clearly state this. If you use a theorem, clearly state its name and show that its hypotheses are satisfied.

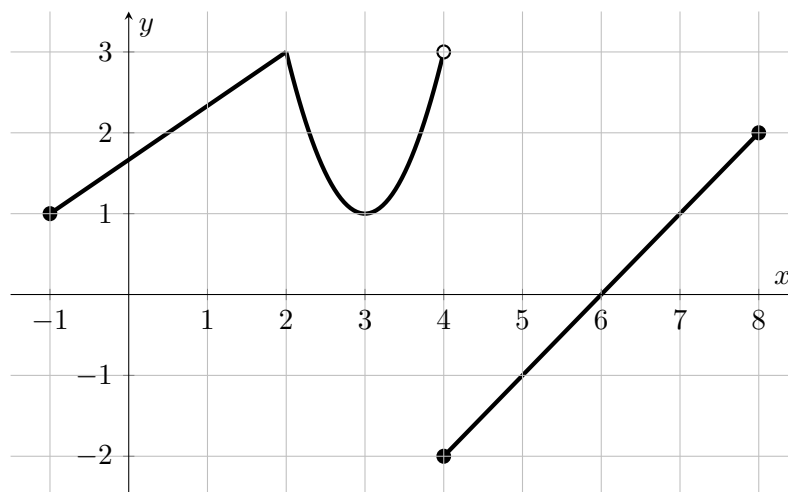
(Reminder: You may not use L'Hopital's Rule or Dominance of Powers in any solutions on this exam.)

(a) $\lim_{x \rightarrow -2} \frac{|x^2 + 5x + 6|}{x + 2}$

(b) If $4x^3 - 6x^2 \leq f(x) \leq x^4 - 4x + 1$ for all x , evaluate $\lim_{x \rightarrow 1} f(x)$.

(c) $\lim_{y \rightarrow \pi/4} \left(\frac{1 - \tan y}{\sin y - \cos y} \right)$

3. (12 points) The function $s(x)$ is graphed in its entirety below. It consists of two line segments and a portion of a parabola. Use the graph to answer the questions below. For this problem, no justification is required for your final answers.



(a) What is the domain of $r(x) = \frac{1}{s(x)}$? State your answer using interval notation.

(b) On what interval(s) is $r(x) = \frac{1}{s(x)}$ continuous? State your answer using interval notation.

(c) What is the domain of $v(x) = \sqrt{s(x)}$? State your answer using interval notation.

(d) Provide a complete formula for $s(x)$ as a piecewise-defined function.

4. (18 points) Consider the function $g(x) = \frac{\sin x}{x(x - \pi/2)}$.

- (a) Identify all values of x , if any, for which $g(x)$ has a vertical asymptote. Justify your answer by evaluating the appropriate limit(s).
- (b) For what value(s) of a is the following piecewise function $h(x)$ continuous at $x = 0$? Justify your answer using the definition of continuity.

$$h(x) = \begin{cases} g(x) & , x \neq 0, \frac{\pi}{2} \\ a & , x = 0 \end{cases}$$

5. (16 pts) A major movie studio has found that the function

$$P(t) = \frac{3t^3 - 9t}{\sqrt{4t^6 + 5t^4 + 5t}}$$

models their profit (in millions of dollars) from a certain movie $t > 0$ weeks after it was released.

- (a) How many weeks after it is released does the movie studio “break even” on the movie? That is, when does $P(t) = 0$ for a realistic value of t ?
- (b) How much profit does the movie studio make in the long run? In other words, what is $\lim_{t \rightarrow \infty} P(t)$? Use correct units in your final answer.
(Reminder: You may not use L'Hopital's Rule or Dominance of Powers in any solutions on this exam.)
6. (10 points) Correctly use a theorem to determine a closed interval in which $x + \tan x = 1$ has a solution. (Be sure to state the name of the theorem that is used and to clearly show that its hypotheses are satisfied.)