APPM 1350
Exam 3
Fall 2022

Name	
Student ID	
Instructor	**Lecture Section**

This exam is worth 100 points and has **5 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

Show all work and ***simplify*** **your answers.** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

End of Exam Check List

1. If you finish the exam before 7:45 PM:
   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors.
2. If you finish the exam after 7:45 PM:
   - Please wait in your seat until 8:00 PM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors.

Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad \cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

1. (32 pts) The following problems are unrelated. For each, be sure to fully simplify your answers.

   (a) Evaluate $\displaystyle\int \frac{t^2+2}{t^{1/3}}\,dt$.

   (b) Evaluate $\displaystyle\int_{\pi^2/4}^{\pi^2} \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$.

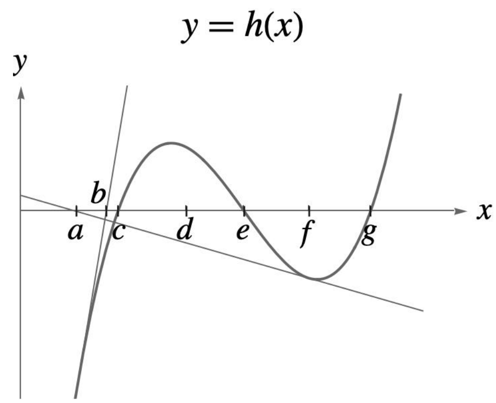
   (c) Suppose

   - $\displaystyle\int_0^2 r(x)\,dx = 6$,
   - $\displaystyle\int_{-1}^0 r(x)\,dx = 5$, and
   - $r(x)$ is an odd function.

   Use this information to evaluate $\displaystyle\int_{-2}^{-1} r(x)\,dx$.

   (d) Evaluate the following sum: $\displaystyle\sum_{i=1}^{n}\left[\left(2+\frac{i}{3n}\right)\frac{3}{n}\right]$. (Your final answer will be in terms of n but will not have any sigmas present.)

2. (8 pts) Suppose Newton's Method is applied to the function $h(x)$, shown below, starting with an **initial approximation of x_1.** Tangent lines to the curve $y = h(x)$ corresponding to the first two iterations of Newton's Method are shown. No justification is necessary for the following two problems.



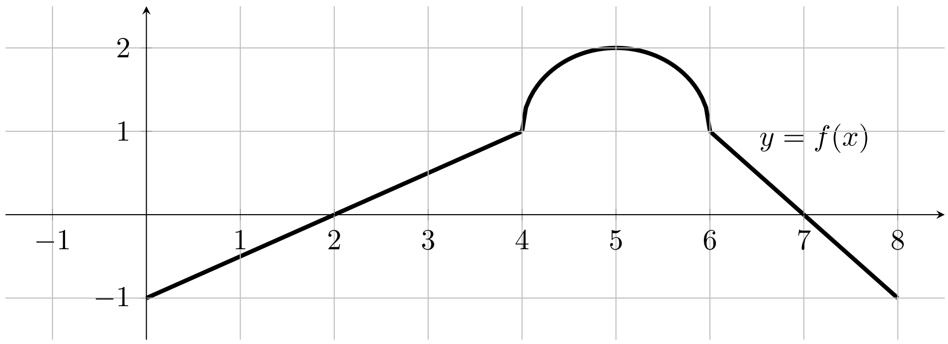
(a) Match the first three approximations x_1, x_2, and x_3 to the x-coordinates a, b, c, d, e, f, or g. Enter your answers in the table below.

Approximation	a, b, c, d, e, f, or g
x_1 (initial guess)	
x_2	
x_3	

(b) Assume that if Newton's Method continues, it will converge. To what value will it converge? Circle your answer in the space provided here. Circle only one answer.

$a \qquad b \qquad c \qquad d \qquad e \qquad f \qquad g$

3. (24 pts) Consider the function $f(x)$ defined over $[0,8]$ that is graphed below. It consists of two line segments and a semicircle.



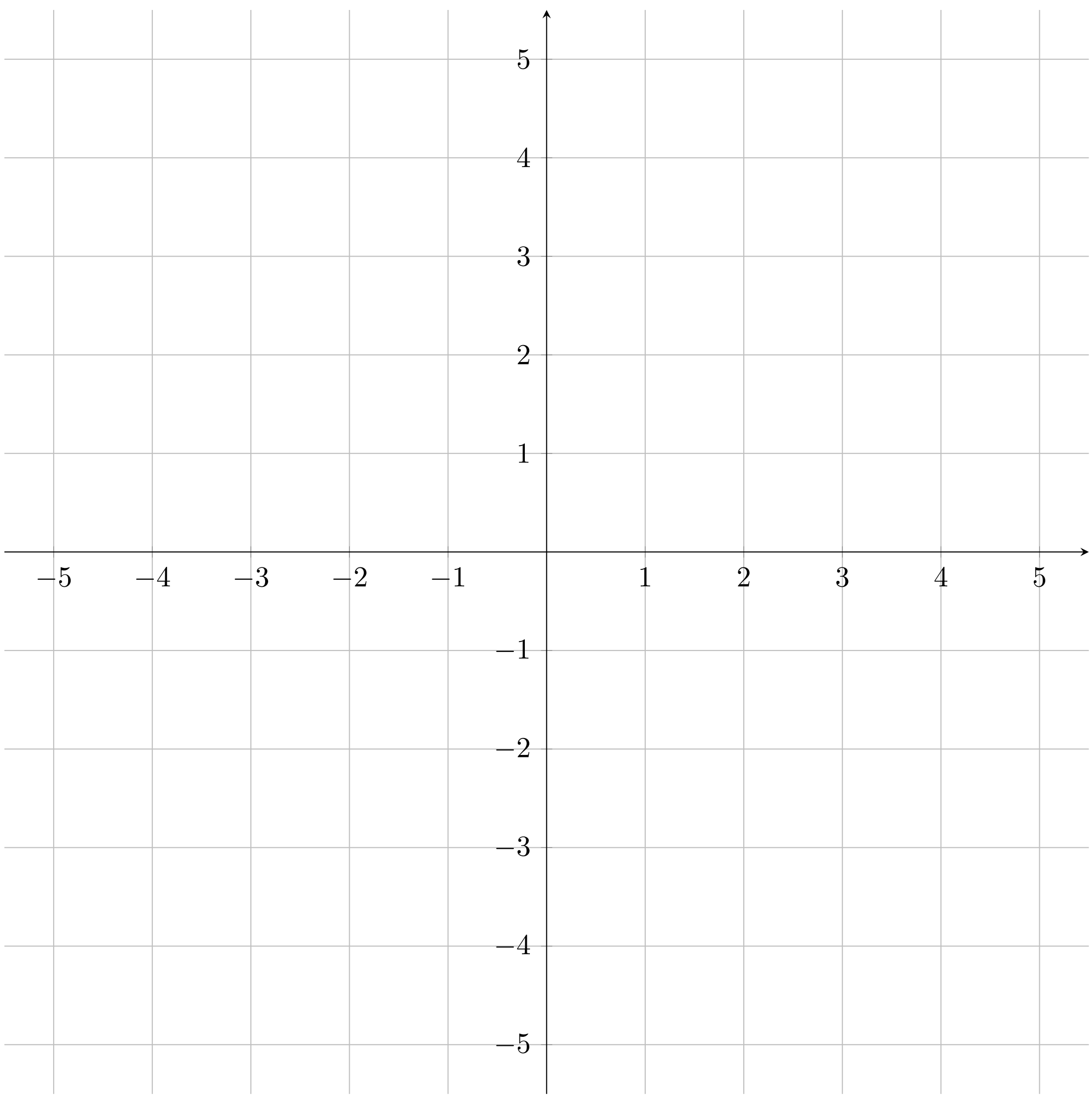
(a) Find the average value of f over $[0,8]$. (Express your answer in terms of π.)

(b) Approximate $\displaystyle\int_2^6 f(x)\,dx$ using four rectangles of equal width with the right endpoint rule.

(c) Let $g(x) = \displaystyle\int_0^x f(t)\,dt$. Find the x-coordinates of all inflection points of $g(x)$. (Provide a brief explanation.)

4. (15 pts) Using the grid below, sketch the graph of a single function, $y = s(x)$ with each of the following characteristics. (Sketch dashed lines to indicate any asymptotes that are present. The concavity of your graph should be clear.)

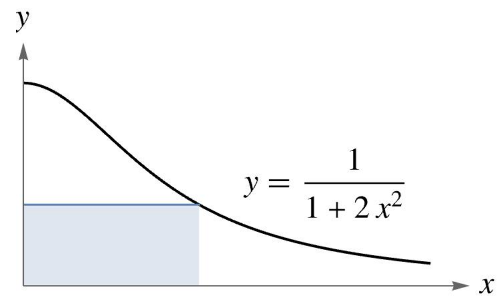
- s is continuous for all $x \neq 3$
- $\lim_{x \to 3^+} s(x) = \infty$
- $\lim_{x \to 3^-} s(x) = -\infty$
- $s''(x) < 0$ only for x in $(-\infty, 3)$
- $s(0) = 0$
- $\lim_{x \to \infty} (s(x) - (2x - 4)) = 0$



5. (21 pts) The following problems are unrelated.

(a) A particle has acceleration given by $a(t) = 4t - 10$ meters per second squared at t seconds. It has a velocity of 4 meters per second at $t = 1$ seconds. Given this, at what time(s) is the particle at rest? Be sure to fully justify your answers.

(b) The rectangle shown has one side on the positive x-axis, one side on the positive y-axis, and its upper right corner on the curve $y = \dfrac{1}{1+2x^2}$. What dimensions give the rectangle its largest area? (Be sure to justify you have found the absolute maximum.)



END OF TEST

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.