This exam is worth 100 points and has 4 problems.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space at the end of the test. Be sure to make a note indicating the page number where the work is continued or it will not be graded.

Show all work and simplify your answers. Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise. Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

End of Exam Check List

1. If you finish the exam before 7:45 PM:
   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors.

2. If you finish the exam after 7:45 PM:
   - Please wait in your seat until 8:00 PM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors.

Formulas

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$
$$\cos(2\theta) = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$
1. (30 pts) The following problems are unrelated.

(a) Find the tangent line of \( y = \frac{x}{x + 1} \) at \( x = 1 \).

(b) Find the derivative of \( p(x) = \sin^2(x) + \sec(x^3) \). (Please do not simplify your final answer.)

(c) Find \( \frac{dy}{dx} \) at \( (0, 1) \) when \( (x - y)^5 = (x - 1)y^3 \).
2. (22 pts) The following problems are unrelated.

(a) Consider \( r(x) = \tan(x) \).

(i) Determine the linearization of \( r(x) \) at \( x = \frac{\pi}{4} \).
(ii) Use your answer from (i) to approximate \( \tan(1) \). (Your final answer should be in terms of \( \pi \).)
(iii) Is your approximation from (ii) an overestimate or an underestimate? Explain your answer.

(b) Suppose \( f \) is a function where the following are all true:

- \( f'(x) = \frac{x^2}{1 + 2f(x)} \).
- \( f(1) = 2 \), and
- \( f'(x) \) is differentiable at \( x = 1 \).

Using this information, determine \( f''(1) \).
3. (28 pts) The following problems are unrelated.

(a) Use the definition of the derivative to show that the derivative of \( g(x) = 5x^2 - 4x \) is \( g'(x) = 10x - 4 \).

(b) Consider \( s(x) = (x - 3)^{-1} \). Show there is no value \( c \) in \((0, 6)\) such that \( s'(c) = \frac{s(6) - s(0)}{6} \). Why does this not contradict the Mean Value Theorem?

(c) Consider \( r(x) = x(x + 8)^{\frac{1}{3}} \). Determine all critical numbers of \( r(x) \).
4. (20 pts) The following problems are unrelated.

(a) A gloopy is an intelligent species that lives on the planet Blorpy. A gloopy’s volume, $V$, is always the product of their wingspan squared, $W^2$, and their foot length, $F$:

$$ V = W^2 F. $$

Additionally, a gloopy’s foot length grows at a constant rate of 2 cm/year, and their wingspan grows at half that rate. How quickly is the volume of a gloopy increasing if its current foot length is 6 cm and its wingspan is 20 cm?

(b) Graphed below are $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Match each of these to the correct label of A, B, or C. Place your answer in the table below. (No justification is required.)

```
<table>
<thead>
<tr>
<th>Function</th>
<th>A, B, or C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = f(x)$</td>
<td></td>
</tr>
<tr>
<td>$y = f'(x)$</td>
<td></td>
</tr>
<tr>
<td>$y = f''(x)$</td>
<td></td>
</tr>
</tbody>
</table>
```
If you write a solution here, please clearly indicate the problem number.