

APPM 1350

Final Exam

Fall 2021

Name	
Student ID	
Instructor	Section

This exam is worth 150 points and has **7 problems**.

**Show all work and simplify your answers.** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

### Formulas

$$\sum_{i=1}^n i = \frac{n(n+1)}{2} \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sin(2x) = 2 \sin(x) \cos(x) \quad \cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin^2(x) = \frac{1}{2} (1 - \cos(2x)) \quad \cos^2(x) = \frac{1}{2} (1 + \cos(2x))$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) + C \quad \int \frac{dx}{1+x^2} = \tan^{-1}(x) + C \quad \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}(x) + C$$

1. (24 pts) Consider

$$f(x) = \begin{cases} 2x + 1 & 0 \leq x \leq 1 \\ 4 - x^2 & 1 < x \leq 2 \end{cases}$$

- Sketch a graph of  $y = f(x)$ . (The axes should be clearly labeled.)
- Is  $f$  differentiable at  $x = 1$ ? (Justify your answer with the definition of the derivative.)
- Does the Mean Value Theorem apply to  $f(x)$  on  $[0, 2]$ ? Why or why not? (Note: A complete answer will include a statement of the Mean Value Theorem and which hypotheses are satisfied or not.)
- Evaluate  $\int_0^2 f(x) dx$

2. (22 pts) The following problems are not related.

- The planet N'Var has acceleration due to gravity of -12 meters per second squared. Suppose a stone is thrown from the top of a 42 meter tall building on the planet N'Var with an initial upward velocity of 36 meters per second. What is the velocity of the stone when it strikes the ground?

- (b) Determine the absolute minimum and the absolute maximum of  $f(x) = e^{x^3+3x^2}$  over  $[-5, -1]$ .
3. (20 pts) Let  $y(x) = \left(1 + \frac{1}{x^2}\right)^x$
- Find the value of  $y'(1)$ .
  - Evaluate  $\lim_{x \rightarrow \infty} y(x)$ .
4. (28 pts) The following problems are not related.
- Find the derivative of  $y = x^2 \arctan(e^{5x})$ .
  - Evaluate  $\int_0^{\frac{\sqrt{3}}{4}} \frac{1}{\sqrt{1-4x^2}} dx$ .
  - Evaluate  $\int_1^2 \frac{\sinh(\ln x)}{x} dx$ .
5. (24 pts) The following problems are not related.
- Solve the following inequality:  $\log_{10}(4 - 3x) < 1$ . State your final answer using interval notation.
  - Suppose that  $f(5) = 1$ ,  $f'(5) = 6$ ,  $g(5) = -3$ , and  $g'(5) = 2$ . Find  $(f/g)'(5)$ .
  - Evaluate  $\lim_{x \rightarrow 0} \frac{1}{x} \int_0^x \sin(t^2 + 1) dt$
6. (16 pts) A rectangle is growing, but its length is always twice its width. Initially, the width of the rectangle is 3 centimeters. Two minutes later the width is 5 centimeters. If the rate of change of the width is proportional to the width (that is,  $\frac{dw}{dt} = kw$ ), find the rate of change of the area of the rectangle after ten minutes.
7. (16 pts) Sketch a graph of a single function  $y = f(x)$  with all of the following properties:
- The domain of  $f$  is  $(-3, 3)$ .
  - $\lim_{x \rightarrow a} f(x) = f(a)$  for all  $a$  except  $a = 0$
  - $f(1) = 0$
  - $f(-2) = 0$
  - $\lim_{x \rightarrow 3^-} f(x) = \infty$
  - $\lim_{x \rightarrow 0^-} f(x) = -\infty$
  - $f''(x) > 0$  for  $x$  in  $(0, 3)$
  - $f$  is an odd function

END OF TEST
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