APPM 1350
Exam 3
Fall 2021

Name	
Student ID	
Instructor	**Section**

This exam is worth 100 points and has **4 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space on page 5 and at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

Show all work and simplify your answers. Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

Formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

1. (40 pts) The following problems are not related.

   (a) Evaluate $\displaystyle\int_1^9 \frac{r+3}{\sqrt{r}}\,dr$.

   (b) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \tan^5 x \sec^2 x\,dx$.

   (c) Evaluate $\displaystyle\int_0^2 [x^3 - 2f(x)]\,dx$ where $\displaystyle\int_0^2 f(x)\,dx = 6$.

   (d) Evaluate the sum: $\displaystyle\sum_{i=1}^{40} 5(i-1)^2$. (You do not need to simplify your final answer, but it should be in a form that could be directly input into a calculator.)

   (e) If the average value of h on $[-2, 6]$ is 4, then evaluate $\displaystyle\int_{-2}^{6} h(x)\,dx$.

2. (24 pts) Consider the function $f(x) = x - \cos x$.

   (a) Estimate the location of the x-intercept of $f(x)$ by applying one iteration of Newton's method with an initial approximation of $x_0 = \pi/6$. Fully simplify your result.

(b) Use the Right Endpoint Rule with $n = 3$ to approximate the value of $\int_0^{\pi} f(x)\,dx$.

(c) Find the derivative with respect to x of $g(x) = \int_{x^2+1}^{0} f(t)\,dt$.

3. (14 pts) Suppose that you want to build a cylindrical water tank with 9π ft^3 capacity. The cost of building each square foot of wall is \$1, each square foot of the bottom base costs \$6 and building each square foot of the top costs \$3. Find the dimensions that will minimize the cost of building the tank.

4. (22 pts) Let $g(x) = \dfrac{x^2 - 9}{x^2 + 9}$ and define $A(x) = \int_{-7}^{x} g(t)\,dt$.

   (a) On which interval(s) is A increasing? Decreasing?

   (b) On which interval(s) is A concave up? Concave down?

   (c) Draw a graph of $A(x)$ for $-7 \leq x \leq 7$ that clearly shows the x-coordinates of local extrema, inflection points, and intercepts. Assume $\int_{-7}^{0} g(x)\,dx = 0$ to determine the intercepts. (This is a close approximation, but you may assume it is exact for the purposes of your graph.)