

## 1. (24 pts) Trigonometry

- (a) A plane takes off in a straight line with an angle of inclination of  $\pi/3$ . How far (in the horizontal direction) will the plane have traveled when it reaches a height of 12 km?
- (b) Let  $f(\theta) = 6 \cos(2\theta) - 1$
- Sketch a graph of  $f$  on the interval  $[0, 2\pi]$ . (On your graph, label the coordinates of the  $y$  intercept and the coordinates of the maximum and minimum values of the function.)
  - What is the range of  $f$ ?
  - Solve the inequality  $6 \cos(2\theta) - 1 > 2$ . for  $\theta$  in the interval  $[0, 2\pi]$ . Write your answer in interval notation.

2. (26 pts) **Limits** Evaluate the following limits and simplify your answers.  
(Reminder: You may not use L'Hopital's Rule in your solution.)

- (a)  $\lim_{x \rightarrow 3} \frac{|x - 3|}{2x^2 - 5x - 3}$
- (b)  $\lim_{x \rightarrow 0} \frac{\tan(3x) \cos(4x)}{x}$
- (c)  $\lim_{x \rightarrow -\infty} \left( \sqrt{x^2 + 3x} - \sqrt{x^2 - 3x} \right)$

3. (26 pts) Consider the function  $f(x) = \frac{3x^2 + 6x - 9}{x^2 - 3x + 2}$ .

- (a) What is the domain of  $f$ ?
- (b) Find the equation of each vertical asymptote of the function  $y = f(x)$ , if any exist. Support your answer by evaluating the appropriate limits.
- (c) Determine the equation of each horizontal asymptote of the function  $y = f(x)$ , if any exist. Support your answer by evaluating the appropriate limits.
- (d) Use the definition of continuity and your work in parts (a)-(c) to identify the values of  $x$  at which  $f(x)$  is discontinuous. Describe the type of discontinuity at each value.

## 4. (24 pts) The following problems are not related.

- (a) Suppose  $f$  and  $g$  are both odd functions and  $h$  is an even function. Furthermore, suppose  $f$ ,  $g$ , and  $h$  are all defined for all real numbers. Let  $j(x) = h(f(x) + g(x))$ . Determine if  $j(x)$  is even, odd, or neither.
- (b) Consider  $s(x) = \frac{x}{1-x}$  and  $r(x) = \frac{x}{x+1}$ . Determine  $s \circ r$  and simplify as much as possible. Also, determine the domain of  $s \circ r$ .
- (c) Use a theorem to show that  $\cos x = \frac{1}{x}$  has a solution on one of the following intervals:
- $[\frac{\pi}{2}, \pi]$
  - $[-\frac{\pi}{2}, \frac{\pi}{2}]$
  - $[-\pi, -\frac{\pi}{2}]$

(Be sure to state the name of the theorem that is used and justify its use. Note that you only need to show a solution exists on one of the intervals, and that some of them may not work. Be sure to identify the correct interval.)