

This exam is worth 150 points and has 7 questions.

**Start each problem on a new page and put your name at the top of each page.**

Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified. Name any theorem that you use and explain how it is used. Answers with no justification will receive no points unless the problem explicitly states otherwise

Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to operate your camera for proctoring, view the exam, contact your proctor, or to upload your work.

When you have completed the exam, send a message through chat to your proctor. Your proctor will then give you the ok to scan your exam and upload it to Gradescope. **Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave Proctorio or the zoom proctoring room!**

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1. (38 points) Find the requested information. Justify your answer.

(a) Simplify  $\cos^{-1}(\cos(5\pi/4))$

(b) Simplify  $\tan(\sin^{-1}(2x))$

(c) Write the following sum in sigma notation:  $2 - \frac{3}{8} + \frac{4}{27} - \frac{5}{64} + \frac{6}{125} - \frac{7}{216}$ .

(d)  $\lim_{x \rightarrow 0} \frac{x7^x}{7^x - 1}$

(e) Find  $dy/dx$  for  $y = (\ln x)^{\sin 3x}$  (Find  $dy/dx$  in terms of  $x$ , but you need not simplify your answer further.)

2. (20 points) Evaluate the following integrals. Simplify your answer.

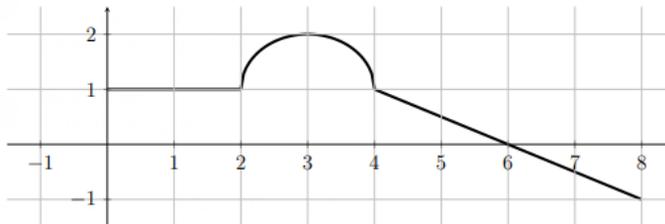
(a)  $\int_0^{\ln 2} \tanh x \, dx$

(b)  $\int_1^{3^{1/4}} \frac{t}{1+t^4} \, dt$  (Note: If you find the upper limit of this integral hard to read, it is 3 raised to the  $1/4$  power.)

3. (30 points) Three unrelated problems. Explain your work and state any theorem that you use.

(a) Suppose that  $2 \leq g'(x) \leq 5$  is true for all  $x$  and that  $g(0) = a$ . Find a reasonable upper and lower bound for  $g(4)$ . Briefly explain your reasoning.

- (b) Find an upper and lower bound for  $\int_{-3}^{-1} \frac{t^2}{1+t^2} dt$  by finding the upper Riemann sum and the lower Riemann sum with  $n = 1$ .
- (c) Let  $f(x) = \int_1^x \frac{t^2}{t^2 + t + 2} dt$  and answer the following two questions.
- Find the linearization of  $f$  at  $x = 1$  and use this to approximate  $\int_1^{1.1} \frac{t^2}{t^2 + t + 2} dt$
  - Find the interval(s) where  $f(x)$  is concave down.
4. (15 points) Find the  $(x, y)$  coordinates of the local and absolute maxima and minima of  $f(x) = x\sqrt{1-x}$  on the interval  $[-1, 1]$ . If a local or absolute max or min does not exist, state this.
5. (12 points) You watch the vertical launch of a rocket from 5 km away. Let  $\theta$  be the angle between the ground and your view of the rocket. This angle, called the angle of elevation above the horizon, increases at a rate of  $\pi/30$  rad/sec when  $\theta = \pi/4$ . Find the speed of the rocket at this instant.
6. (15 points) Consider  $f(x) = \frac{e^x}{e^x + 1}$ .
- Show  $f(x)$  is one-to-one.
  - Find the inverse of  $f(x)$ .
7. (20 points) Consider the function  $f(x)$  defined over  $[0, 8]$  that is graphed below. It consists of two straight line segments and a semicircle.



- Find the average value of  $f$  over the interval  $[0, 8]$ .
  - Evaluate  $\lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h}$ .
  - Show that  $h(x) = \frac{f(x)}{(x-6)^2}$  has a vertical asymptote at  $x = 6$ .
8. At the bottom of your work for problem 7: please write a statement that says your work is your own and you did not receive any help on the exam. Sign this statement. **Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave Proctorio or the zoom proctoring room.**