

This exam is worth 100 points and has 5 questions.

Start each problem on a new page and put your name at the top of each page.

Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified. Name any theorem that you use and explain how it is used. Answers with no justification will receive no points unless the problem explicitly states otherwise

Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to operate your camera for proctoring, view the exam, contact your proctor, or to upload your work.

When you have completed the exam, send a message through chat to your proctor. Your proctor will then give you the ok to scan your exam and upload it to Gradescope. **Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave Proctorio or the zoom proctoring room!**

1. (22 points) Find the requested information, if possible. If it is not possible, explain.

(a) Find $s(t)$ if $s'(t) = \sqrt{t} - 3 \sin t$ and $s(0) = 4$.

(b) $\int_{-2}^1 \frac{1}{x^2} dx$

(c) $\int_0^2 \frac{ax^2 + 3b}{(ax^3 + 9bx + 1)^{1/3}} dx$, a and b are positive constants.

2. (17 points)

(a) Find the value of the sum, $\sum_{i=1}^n \left(1 + \frac{3i}{n}\right)^2 \frac{3}{n}$, in terms of n .

(b) Evaluate $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 + \frac{3i}{n}\right)^2 \frac{3}{n}$

(c) What is the definite integral that corresponds to the quantity in part (b)?

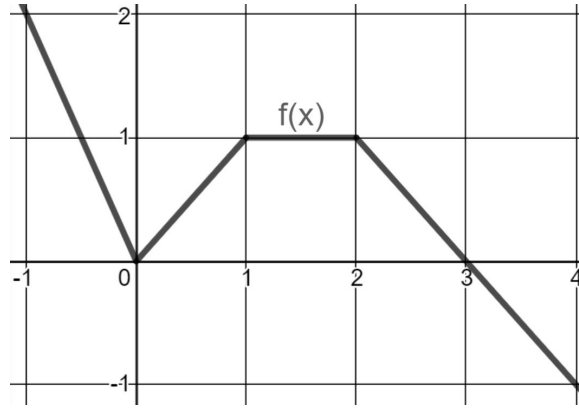
3. (21 points) Unrelated, short-answer questions:

(a) Without trying to compute the integral, provide a reasonable upper bound and a reasonable lower bound for $\int_{-2}^{-1} (1 + t^4)^{1/2} dt$. Briefly explain your reasoning.

(b) Find the slant asymptote for $g(x) = \frac{x^3}{x^2 + x - 9}$.

(c) Estimate $(17)^{1/4}$ by using Newton's method on $f(x) = x^4 - 17$ as follows: Do one iteration with $x_0 = 2$. What is x_1 ?

4. (16 points) You plan to make a box with a square base. The volume of the box will be 20 cubic inches. The material for the bottom of the box costs \$8 per square inch. The material for the sides of the box cost \$4 per square inch. The material for the top of the box costs \$12 per square inch. What are the dimensions of this box that will minimize the cost of the materials?
5. (24 points) For this problem, consider the graph of the function $f(x)$, continuous for all x and shown below on the interval $[-1, 4]$. For parts (a) - (d) find the requested information.



- (a) Find the upper Riemann sum for f on the interval $[-1, 4]$ using 5 equally spaced subintervals.
- (b) Find the average value of f on the interval $[-1, 4]$.
- Now, let $p(x) = x + \int_0^{x^2} f(t) dt$ with f still given by the figure above.
- (c) Find $p(1)$.
- (d) Find $p'(2)$.
6. At the bottom of your work for problem 5: please write a statement that says your work is your own and you did not receive any help on the exam. Sign this statement. **Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave Proctorio or the zoom proctoring room.**

Midterm 3 formulas:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2} \right)^2$$