APPM 1350 **Exam 2** **Fall 2020**

This exam is worth 100 points and has 4 questions.

Start each problem on a new page and put your name at the top of each page.

Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified. Name any theorem that you use and explain how it is used. Answers with no justification will receive no points unless the problem explicitly states otherwise

Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to operate your camera for proctoring, view the exam, contact your proctor, or to upload your work.

When you have completed the exam, send a message through chat to your proctor. Your proctor will then give you the ok to scan your exam and upload it to Gradescope. **Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave Proctorio or the zoom proctoring room!**

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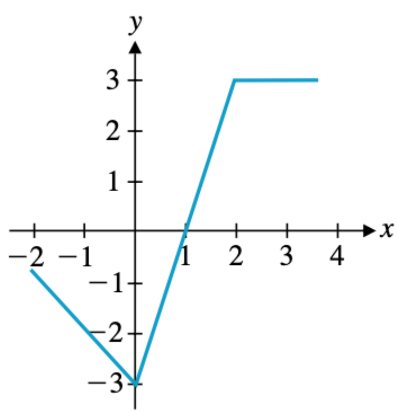
1. (38 points) Find the requested information.

   (a) Find $dg/d\theta$ for $g(\theta) = \theta \sin(c^2\theta^2 + 3)$ and c a positive constant. Do not simplify your final answer.

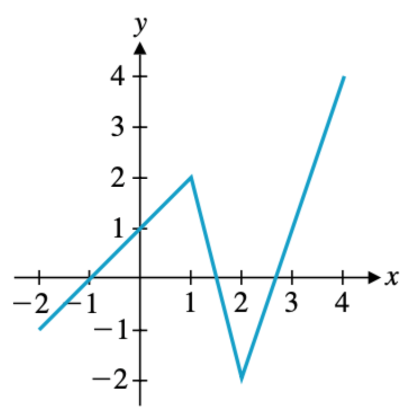
   (b) Find df/dt for $f(t) = \dfrac{t^2 - 3t}{t^{2/3} + 5}$. Do not simplify your final answer.

   (c) Find the equation of the line tangent to $y^2 + \sin(xy) = 4$ at the point $(0, 2)$.

   (d) The graphs of $y = f(x)$ and $y = g(x)$ are given below.



$y = f(x)$



$y = g(x)$

   Find the derivative of $g(f(x))$, if it exists, at $(i) x = 0$, $(ii) x = 1$, and $(iii) x = 3$. If the derivative does not exist, state this and explain why it does not exist.

2. (15 points) A conical tank (i.e. an inverted cone) of height $h = 10$ meters and base radius $r = 4$ meters is full of water. The water drains from the bottom of the tank at the rate of 5 m^3/min. How fast is the water level, h, dropping when $h = 6$ meters? (Note: The volume of a cone is given by $V = (1/3)\pi r^2 h$ where r is the radius of the base of the cone and h is the height.)

3. (20 points) Let $g(x) = 6\sqrt{x} - 2x,\ x \geq 0.$

   (a) Use the definition of the derivative to show that $g'(x) = \dfrac{3}{\sqrt{x}} - 2$. (You must use the limit definition of the derivative to earn any credit on this problem.)

   (b) Verify that the hypotheses of the Mean Value Theorem are satisfied for g(x) over [0,16]. Then, determine the value(s) c that satisfy the conclusion of the Mean Value Theorem.

4. (27 points) Consider the function $f(x) = x^{\frac{1}{3}}(x+4)$. It has first derivative $f'(x) = \dfrac{4(x+1)}{3x^{\frac{2}{3}}}$ and second derivative $f''(x) = \dfrac{4(x-2)}{9x^{\frac{5}{3}}}$. The following questions will take you step by step through the information you need to sketch a graph of the function.

   (a) On what intervals is f increasing? decreasing?

   (b) Find the x and y coordinates of the local maximum and minimum values of f, if any exist. If none exist, state this.

   (c) On what intervals is f concave up? concave down?

   (d) Find the x and y coordinates of the inflection points of f, if any exist. If none exist, state this.

   (e) Sketch a graph of f. Carefully label all important points (intercepts, max/min, inflection point(s), etc.)