
This exam is worth 100 points and has 4 questions.

Start each problem on a new page and put your name at the top of each page.

Show all work and simplify your answers. Name any theorem that you use and explain how it is used. Answers with no justification will receive no points unless the problem explicitly states otherwise

Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to operate your camera for proctoring, view the exam, contact your proctor, or to upload your work.

When you have completed the exam, send a message through chat to your proctor. Your proctor will then give you the ok to scan your exam and upload it to Gradescope. Verify that everything has been uploaded correctly and pages have been associated to the correct problem before you leave the zoom proctoring room!

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1. (26 points) The following three problems are not related.
   (a) Suppose $\tan\theta = \frac{5}{2}$ and $\pi < \theta < \frac{3\pi}{2}$. What is the value of $\sec\theta$?
   (b) Find the solutions of $\sin^2 t + 2\sin t = 3$ in $[0, 2\pi)$.
   (c) You are standing some distance away from a tree. The angle of elevation (from the ground where your feet are, to the top of the tree) is $\pi/3$ radians.
      i. How far away are you from the tree if the tree's height is $3\sqrt{3}$ meters?
      ii. What is the distance from your feet to the top of the tree?

2. (24 points) Evaluate the limits if they exist. If the limit does not exist, write DNE. Be sure to explain all work. If you use a theorem, please state this and explain how you used it.
   (a) $\lim_{x\to 2} \dfrac{|x-2|}{x^4 - 4x^2}$
   (b) $\lim_{x\to\infty} (\sqrt{x^2 - x - 1} - x)$
   (c) $\lim_{x\to 0} (x^2\cos(1/x) + 5)$

3. (20 points) Suppose $y = f(x)$ is a function with domain all real numbers.
   (a) Give a short explanation (one or two sentences) of what each of the following conditions tell you about a graph of $y = f(x)$.
      i. $\lim_{x\to\infty} f(x) = 3$ and $\lim_{x\to-\infty} f(x) = -3$
      ii. $\lim_{x\to 2^+} f(x) = -\infty$ and $\lim_{x\to 2^-} f(x) = \infty$
   (b) If you also know that $y = f(x)$ is **odd** and if you know that $f(2) = 1$ and $f(4) = 0$, sketch a graph of $y = f(x)$ that satisfies this information and the information given in part (a). Label the axes, asymptotes, and any x or y intercepts.

4. (30 points) The following three questions are unrelated:

   (a) Let $f(x) = x + \dfrac{1}{x}$ and $g(x) = \dfrac{3x+5}{x+2}$. Find $(g \circ f)(x)$ and the domain of $(g \circ f)(x)$. Give the domain in interval notation.

   (b) Use the definition of continuity (i.e. use the appropriate limits) to find all values of a which make $f(x)$ continuous everywhere.

$$f(x) = \begin{cases} x^2 + 4 & x \geq a \\ 4x & x < a \end{cases}$$

   (c) Show that $t^3 + \sin t = \frac{1}{t^2+1}$ has at least one solution. If you use a theorem, state this and explain how you used it.

5. At the bottom of your work for problem 4: please write a statement that says your work is your own and you did not receive any help on the exam. Sign this statement.

Trigonometric identities

$\sin(2x) = 2\sin(x)\cos(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$

$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$