APPM 1350 **Final Exam** **Fall 2019**

On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 150 points and has 7 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

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1. (32 pts) Evaluate the following. Simplify your answers.

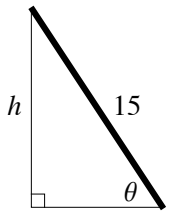
   (a) $\lim_{x\to 0} \dfrac{5x^{40} - 8x^{20}}{10x^{40} + 7x^{20}}$

   (b) $\lim_{x\to 0^+} (\sin x)^{\pi/(\ln x)}$

   (c) Let $f(t) = \arctan\left(3^t\right)$. Find $f'(1)$.

   (d) Let $g(t) = (1+2t)^{\cos t}$. Find $g'(\pi)$.

2. (12 pts) A ladder 15 ft long propped against a wall forms an angle of θ radians with the ground. The top of the ladder begins to slide down the wall at a rate of $\frac{1}{4}$ ft/sec. How fast is θ changing when the top of the ladder is $h = 12$ ft above the ground?



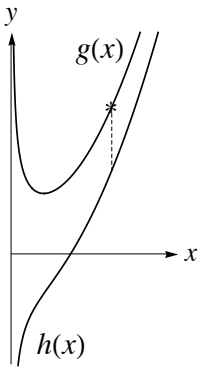
3. (24 pts) Evaluate the following. Simplify your answers.

   (a) $\displaystyle\int_{1/2}^{1} 8x^{-2}\left(1+\frac{1}{x}\right)^{-3} dx$

   (b) $\displaystyle\int \frac{s^2}{\sqrt{1-s^6}}\, ds$

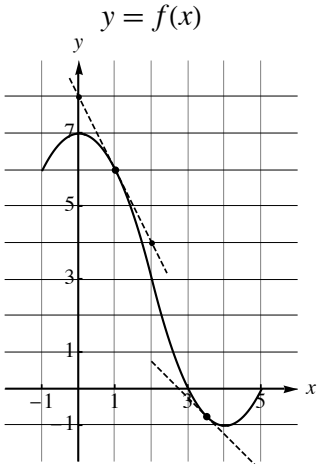
   (c) Let $f(x) = 10 + \displaystyle\int_9^{x^2} \sin\left(\frac{\pi\sqrt{t}}{2}\right) dt$. Find $f'(3)$.

4. (12 pts) Buff Bug is crawling along the curve defined by $g(x) = x^2 + \dfrac{1}{\sqrt{x}}$, $x > 0$, and wishes to move down to a lower curve $h(x) = x^2 - \dfrac{x}{8} - \dfrac{1}{\sqrt{x}}$ using the shortest vertical route possible. At what x-coordinate should Buff Bug descend?



TURN OVER—More problems on the back!

5. (36 pts) The continuous and differentiable function $f(x)$, shown at right, is defined for $-1 \le x \le 5$ and has the following properties.



- The tangent line at $x = \frac{7}{2}$ is $y = -\frac{3}{4} - \left(x - \frac{7}{2}\right)$.
- The tangent line at $(1, 6)$ is shown.
- $\displaystyle\int_{-1}^{3} f(x)\,dx = \frac{58}{3}$.
- $\displaystyle\int_{-1}^{5} f(x)\,dx = \frac{54}{3} = 18$.

Answer the following questions about $f(x)$. Justify your work.

(a) Approximate $\displaystyle\int_{-1}^{5} |f(x)|\,dx$ using an upper sum with three equal subintervals.

(b) Find the exact value of $\displaystyle\int_{-1}^{5} |f(x)|\,dx$.

(c) Use the linearization of f at $x = 1$ to approximate $f(0.99)$.

(d) If Newton's Method is applied to $f(x)$ with an initial approximation of $x_1 = \frac{7}{2}$, what is the value of x_2?

(e) Find one of the c values that satisfies the conclusion of the Mean Value Theorem for Derivatives on the interval $[-1, 5]$.

(f) Find the c value that satisfies the conclusion of the Mean Value Theorem for Integrals on the interval $[-1, 5]$.

6. (20 pts) The following two problems are not related.

(a) Find the inverse of the function $y = \dfrac{\cosh x}{e^x}$. (You may assume that y is one-to-one.)

(b) A scientist is working with two radioactive isotopes, A and B. Her two samples of A and B had the same mass m_0 initially but their relative decay rates (constants k_A and k_B, respectively) differ.

i. Find expressions for $m_A(t)$ and $m_B(t)$, the mass of each sample at time t.

ii. The ratio of the two masses, $r(t) = m_A(t)/m_B(t)$, satisfies the differential equation $dr/dt = \alpha r$ for some constant value of α. Find α in terms of k_A and k_B.

7. (14 pts) Sketch a graph of a single function $y = g(x)$ with all of the following properties. No justification is necessary.

- $\lim\limits_{x \to a} g(x) = g(a)$ for all $a \neq -2$
- $\lim\limits_{x \to -2} g(x) = \infty$
- $\lim\limits_{x \to -\infty} g(x) = 2$
- $g(0) = -2$
- $\displaystyle\int_{0}^{2} g(x)\,dx = 0$
- $\displaystyle\lim_{h \to 0} \frac{g(h) - g(0)}{h} = 2$
- $\displaystyle\lim_{x \to a} \frac{g(x) - g(a)}{x - a} < 0$ for $a > 2$