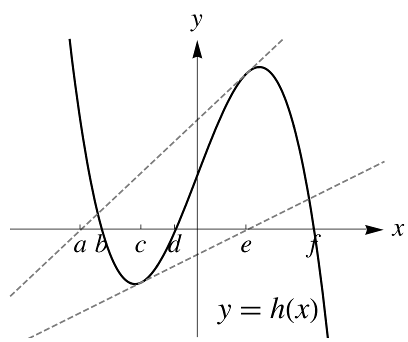
On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

1. (26 pts) The following problems are not related.

   (a) Let $f(x) = \dfrac{2x^3 - 3x^2 + 2x}{x^2 + 1}$. Find the slant asymptote of $y = f(x)$. Justify your answer using limits.

   (b) Suppose Newton's Method is used to approximate a root of $h(x)$. Tangent lines to the curve $y = h(x)$ corresponding to the first two approximations are shown. Match the first three approximations x_1, x_2, and x_3 to the x-coordinates (a, b, c, d, e, or f). No justification is necessary.



   (c) Find the positive integer n that satisfies $\displaystyle\sum_{i=1}^{n} (2i + 4) = 4n + 9900$.

   (d) Suppose g is an odd function continuous on $[-7, 7]$. Given

$$\int_{-3}^{7} g(x)\,dx = 10 \quad \text{and} \quad \int_{-7}^{0} g(x)\,dx = 9,$$

   find $\displaystyle\int_{0}^{3} g(x)\,dx$.

2. (14 pts) Suppose you want to make an open-top box out of a square sheet of cardboard by cutting out a small square from each corner and bending up the sides. If the cardboard sheet measures 1 meter by 1 meter, what is the largest volume that such a box can have?

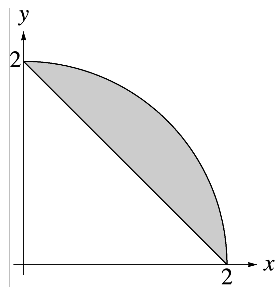
TURN OVER—More problems on the back!

3. (24 pts) The following problems are not related.

   (a) Find $g(\theta)$ given $g''(\theta) = 2\sin\theta + \cos\theta$, $g(0) = \pi$, and $g\left(\frac{\pi}{2}\right) = -1$.

   (b) Evaluate $\displaystyle\int x^{-1/2}\sec^5(\sqrt{x})\tan(\sqrt{x})\,dx$.

   (c) Given a circle of radius 2 in the first quadrant, write down an integral (or integrals) to represent the area of the shaded region shown below, then evaluate your integral(s) using geometry.



4. (14 pts) Ralphie travels at a velocity of $v(t) = t^2 - 3t + 2$ starting at $t = 0$ until $t = 2$.

   (a) Find Ralphie's average velocity on $[0, 2]$.

   (b) Sketch a graph of $v(t)$ and mark the approximate location of the number c that satisfies the Mean Value Theorem for Integrals. Justify your answer. (It is not necessary to find the exact value of c.)

5. (22 pts) The following two problems are not related.

   (a) Consider the curve $y = \dfrac{x}{(x^2+3)^2}$, $1 \le x \le 3$.

      i. Find an expression for R_n, the Riemann sum using right endpoints and n equal subintervals. Do not evaluate the expression.

      ii. Find $\displaystyle\lim_{n\to\infty} R_n$ by evaluating an integral.

   (b) Let

$$g(x) = 2 + \int_{x^2}^{4} \cos\left(\frac{\pi\sqrt{t}}{2}\right) dt, \quad 0 \le x \le 3.$$

   Find the linearization of g at $x = 2$.

Formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$