APPM 1350 **Exam 2** **Fall 2019**

On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Limit problems should not be evaluated using L'Hopital's Rule. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

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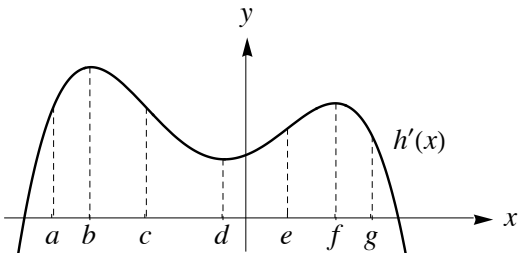
1. (30 pts) The following three problems are not related.

   (a) No justification is necessary for the following questions about the graph of derivative $h'(x)$ shown below. At which of the x-coordinates (a, b, c, d, e, f, or g) is

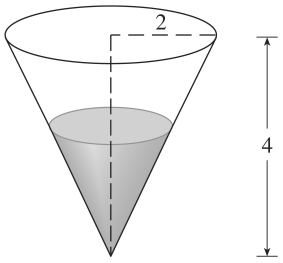
   i. $h(x)$ greatest?
   ii. $h(x)$ least?
   iii. $h'(x)$ greatest?
   iv. $h'(x)$ least?
   v. $h''(x)$ greatest?
   vi. $h''(x)$ least?



   (b) Let $y = \sqrt{x^3 - \sin(\pi x)}$. Find an equation of the line tangent to y at $x = 1$.

   (c) Find dy/dx if $\cos(3xy) = (y-1)^2$.

2. (15 pts) A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being drained from the tank at a rate of π m^3/min, how deep is the water when the water level is falling at 9 m/min?



3. (20 pts) The following two problems are not related.

   (a) Compute an approximation to $\sqrt{1.1}$ by finding a linearization for $g(x) = \sqrt{1+x}$ at $a = 0$.

   (b) When a guitar string is plucked, it vibrates with a frequency

$$f(T) = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

   where T is the tension, L is the length of the string, and μ is the linear mass density. "Tuning" the guitar means adjusting the tension while L and μ stay constant, so that the desired note is produced when plucked. If a musician wants to make sure that her string vibrates so that the relative error in the frequency f is less than 0.01, then what is the largest acceptable relative error in the tension T?

4. (15 pts) Let $f(x) = x + 2\cos(x)$ on $[0, 2\pi]$.

   (a) On what intervals is f increasing? decreasing?

   (b) Find the (x, y) coordinates of the local maximum and minimum values of f, if any.

   (c) Find the (x, y) coordinates of the absolute maximum and minimum values of f.

   $\big($*Hint:* $\sqrt{2} \approx 1.4$ and $\sqrt{3} \approx 1.7\big)$

5. (20 pts) The following two problems are not related.

   (a) Verify that $g(x) = x^3 - 3x^2$ satisfies Rolle's theorem on $[0, 3]$ and find all x-values whose existence is guaranteed by Rolle's theorem on this interval.

   (b) Given

$$f(x) = \begin{cases} x\sin\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0, \end{cases}$$

   use the definition of derivative to determine if $f(x)$ is differentiable at $x = 0$. You may assume that f is continuous at $x = 0$.