APPM 1350 **Exam 1** **Fall 2019**

On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Limit problems should not be evaluated using L'Hopital's Rule. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

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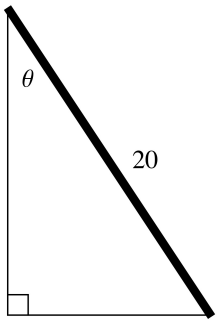
1. (15 pts) The following two problems are not related.

   (a) Solve the inequality on $[0, 2\pi]$ and write your answer in interval notation.

   $$2\cos x \sec x \sin x + 1 < 0$$

   (b) A 20-ft ladder leaning against a wall forms an angle of θ with the wall, with a maximum angle of $\frac{\pi}{3}$ radians. For larger angles the base of the ladder slips and the ladder falls to the ground. What is the slope of the ladder at this maximum angle?



2. (24 pts) Evaluate the following limits and simplify your answers.
   (*Reminder:* You may not use L'Hopital's Rule or derivatives on this exam.)

   (a) $\lim_{t\to 2} \dfrac{2-t}{\sqrt{6}-\sqrt{3t}}$

   (b) $\lim_{x\to 0} x^2 \cos\left(\dfrac{1}{x^2}\right)$

   (c) $\lim_{h\to 0} \dfrac{\sin(\pi+h)-\sin(\pi-h)}{2h}$ (*Hint:* Refer to the formula section.)

3. (20 pts) Consider the function $f(x) = \dfrac{1}{x+1}$.

   (a) Sketch a graph of $y = f(x)$ on $[-3, 3]$. Label all intercepts.

   (b) Suppose we use the precise definition of a limit to verify that $\lim_{x\to 1} f(x) = \frac{1}{2}$.

   i. Illustrate by adding to your graph from part (a). Include labels for δ and ϵ. (You may sketch a new graph instead.)

   ii. Given $\epsilon = \frac{1}{8}$, what is a corresponding value of δ?

   (c) Find $(f \circ f)(x)$ and simplify your answer.

   (d) What is the domain of $f \circ f$? Write your answer in interval notation.

TURN OVER—More problems on the back!

4. (20 pts) Let $g(x) = \dfrac{7x + 21}{3 - |x|}$.

   (a) Express $g(x)$ as a piecewise function without using absolute value.

   (b) Is g continuous at $x = 0$? Justify your answer using the definition of continuity.

   (c) Identify any horizontal asymptotes of the curve $y = g(x)$. Justify your answer using appropriate limits.

5. (21 pts) The following problems are not related.

   (a) Find and classify all discontinuities of $h(x) = \dfrac{2x^2 - 9x + 9}{x^2 - 9}$. Justify using appropriate limits.

   (b) Show that the function $f(x) = \sqrt{x}\cos x$ intersects the line $y = -1$. Based on your work, indicate an interval where the intersection point can be found.

Precise Definition of a Limit

The limit of $f(x)$ as x approaches a is L if for every number $\epsilon > 0$ there is a corresponding number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

Formulas

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$\sin^2(x) = \tfrac{1}{2}(1 - \cos(2x))$$
$$\cos^2(x) = \tfrac{1}{2}(1 + \cos(2x))$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$