
On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 150 points and has 7 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

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1. (a) (16 pts) Evaluate the following limits.

   i. $\lim_{x\to\pi} \dfrac{3^{\sin(x)}-1}{x-\pi}$ ii. $\lim_{x\to\infty} \dfrac{\sinh(x)}{e^x}$

   (b) (16 pts) Consider $f(x) = e^{1/x}$.

   i. Find the domain of the function. Express your answer in interval notation.

   ii. Evaluate $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$.

   iii. Find the horizontal asymptotes of f, if any. Justify using appropriate limits.

2. (30 pts) The following problems are not related.

   (a) The curve given by the equation $x^3 + y^3 = 6xy$ is called the *folium of Descartes*. Find an equation of the tangent line to the folium of Descartes at the point $(3, 3)$. Write your answer in the form $y = mx + b$.

   (b) Let $g(x) = \displaystyle\int_{4/x}^{1} t\cot(t)\,dt$. Find $\dfrac{dg}{dx}$.

   (c) Given $y = (\ln x)^{\cos x}$, find y' at $x = \pi$. Simplify your answer.

3. (12 pts) Two particles are moving along a line. Particle A's position is given by $A(t) = t^2 + 2 + 3\sin^2(t)$. Particle B's position is given by $B(t) = t - 3\cos^2(t)$. At what time are A and B closest?

4. (18 pts) The following problems are not related.

   (a) The sum $S = 11 + 13 + 15 + \cdots + 99$ of odd integers can be written as $\displaystyle\sum_{i=1}^{n} a_i$. Find the value of n and an expression for a_i.

   (b) Approximate the value of $\displaystyle\int_0^1 \cos^{-1}(x)\,dx$ using L_2, the left-endpoint rectangle approximation with two equal subintervals. Simplify your answer.

   (c) Suppose f is a twice-differentiable function and the linearization of $y = x^2 - f(x)$ at $x = 5$ is used to approximate values of y near $x = 5$. Given that $f''(5) = -1$, will the approximations be underestimates or overestimates? Explain.

TURN OVER—More problems on the back!

5. (24 pts) Evaluate the following integrals.

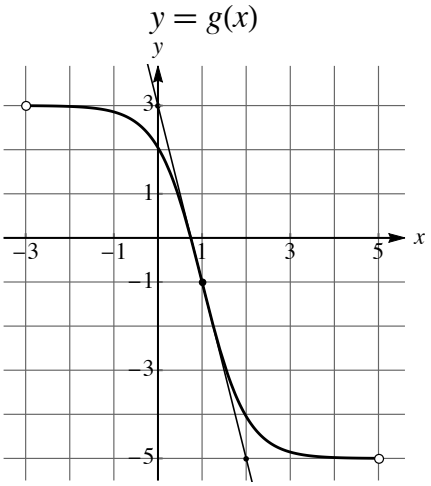
   (a) $\displaystyle\int_0^{10} h(x)\,dx$ given $h(x) = \begin{cases} |x-2| & x \le 6 \\ 4 & x > 6 \end{cases}$.

   (b) $\displaystyle\int \frac{1-t}{\sqrt{9+2t-t^2}}\,dt$

   (c) $\displaystyle\int \frac{\sinh x}{1+\cosh^2 x}dx$

6. (12 pts) A radioactive substance decreases in mass by 17% in 3 years. What is the half-life of the substance?

7. (22 pts)



   Shown above is the graph of $y = g(x)$ defined on $(-3, 5)$, and the line tangent to g at $(1, -1)$.

   (a) Find the value of $\lim_{x\to 0} |g(x) - 5|$.

   (b) Find the value of $\lim_{h\to 0} \dfrac{g(1+h) - g(1)}{h}$.

   (c) Suppose g is the derivative of a function f so that $g(x) = f'(x)$. Estimate the interval(s) on which f is increasing, or write NONE if f is a decreasing function.

   (d) Find the following values for the inverse function of g.

   i. $g^{-1}(2)$ ii. $\left(g^{-1}\right)'(-1)$