

On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

1. **Core Section: Integration** (30 pts)

(a) $\int \frac{\csc^2(1/t)}{t^2} dt$

(b) $\int_0^4 (|x-1| + \sqrt{16-x^2}) dx$

(c) Find the average value of $f(x) = \frac{18x}{(x^2+9)^2}$ on $[0, 3]$.

(d) Find the value of $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(4 - 5 \left(-2 + \frac{2i}{n} \right)^4 \right) \frac{2}{n}$. (Hint: Evaluate an integral.)

2. (15 pts) Beulah Bug is crawling along the curve $y = x^2$. Find the x -coordinates of the points on the curve where Beulah is closest to her home located at $(0, 1)$.

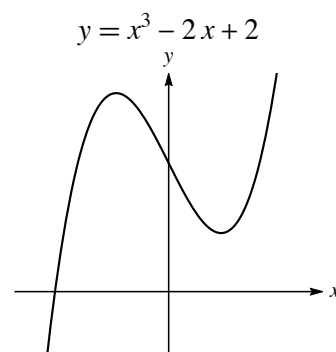
3. (24 pts) The following three problems are not related.

(a) Find an equation for the slant asymptote of $y = \frac{9x^2 + 3x}{3x + 5}$. (It is not necessary to justify your answer with limits.)

(b) If $\sum_{i=1}^n (4i - 2) = 7200$, what is the value of n ?

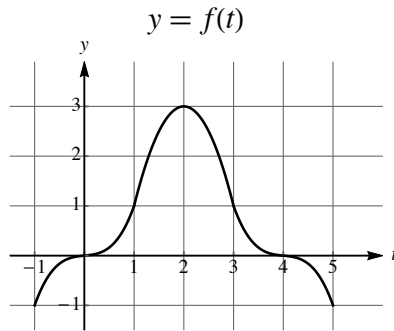
(c) Apply Newton's Method to $y = x^3 - 2x + 2$ with an initial approximation of $x_1 = 0$.

- Calculate the next two approximations x_2 and x_3 .
- Copy the graph of the function. (It is not necessary to draw a precise graph.) Sketch the lines used to find x_2 and x_3 .
- Explain what happens to x_n as $n \rightarrow \infty$. Will x_n approach the root?



TURN OVER—More problems on the back!

4. (15 pts) Ralphie is driving her car at 60 feet per second when she sees a red light ahead, 180 feet away. She immediately brakes with a constant acceleration of $-k$ ft/sec², $k > 0$.
- Find the car's velocity and position functions, $v(t)$ and $s(t)$, in terms of k . Assume that the initial position is 0 feet.
 - How many seconds will it take for Ralphie to come to a complete stop? Express your answer in terms of k .
 - Find the value of k that will cause Ralphie to stop right at the light.
5. (16 pts)



The graph of a differentiable function f on the interval $[-1, 5]$ is shown above. The graph of f has a horizontal tangent line at $t = 2$. Let $g(x) = 5 + \int_2^x f(t) dt$ for $-1 \leq x \leq 5$. No explanations are necessary for the following questions.

- Find the values of $g(2)$, $g'(2)$, and $g''(2)$.
- On what interval(s) is g decreasing?
- On what interval(s) is g concave down?

Formulas

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2} \right)^2$$