
On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Limit problems should not be evaluated using L'Hopital's Rule. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

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1. **Core Section: Differentiation** (30 pts)

   (a) Let $g(t) = \left(at + \sqrt{a^2 + t^2}\right)^{-2}$. Find $\dfrac{dg}{dt}$ and leave your answer unsimplified.

   (b) Given $5xy^3 - 2\sqrt{x^3} = 8y + \sqrt{3}$, find dy/dx. *(Similar to WebAssign 2.6 #5)*

   (c) Find an equation for the line tangent to $y = \dfrac{\sec(3x)}{2 + \tan(3x)}$ at $x = 0$.

2. (15 pts) Let $f(x) = \begin{cases} \frac{8}{3}x & x < 6 \\ (x-2)^2 & x \geq 6 \end{cases}$.

   (a) Write the limit definition of $f'(a)$, the derivative of $f(x)$ at $x = a$.

   (b) Use the definition to determine the value of $f'(6)$ or to show that it does not exist. *(Similar to HW 4 #4b)*

3. (15 pts) A street light is mounted at the top of a 17-ft-tall pole. A woman 5 ft tall walks away from the pole with a speed of 4 ft/s along a straight path. How fast is the tip of her shadow moving when she is 34 ft from the pole? *(Similar to HW 6 #3)*

4. (20 pts) Consider $f(x) = \dfrac{x^2 + x + 4}{x + 1}$ with $f'(x) = 1 - \dfrac{4}{(x+1)^2}$ and $f''(x) = \dfrac{8}{(x+1)^3}$. *(Similar to WebAssign 3.3 #10)*

   (a) On what intervals is f increasing? decreasing?

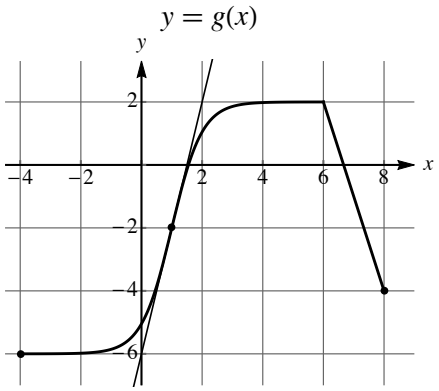
   (b) Find the x and y coordinates of the local maximum and minimum values of f.

   (c) On what intervals is f concave up? concave down?

   (d) Use parts (a) to (c) to sketch a graph of f.

TURN OVER—More problems on the back!

5. (20 pts)



Shown above is the graph of $y = g(x)$ and the line tangent to g at $(1, -2)$. The function g is differentiable on $(-4, 6) \cup (6, 8)$.

(a) Sketch the graph of $y = g'(x)$. Label tick marks clearly. *(Similar to HW 4 #3)*

(b) Use the linearization of g at $a = 1$ to estimate the value of $g(0.7)$. *(Similar to Exam 2 #5 fall 2017)*

(c) The Mean Value Theorem states that there exists a value of c in $(-4, 6)$ such that $g'(c)$ equals a certain value.

i. What is that value?

ii. Suppose we wish to narrow down the possible values for c. Between which two consecutive integers can c be found? List all possible answers. No explanation is necessary.