1. **Core Section: Trigonometry** (20 points)

   (a) Given $\tan\theta = 1/10$ and $\pi < \theta < 3\pi/2$, what is $\cos\theta$? *(Similar to WebAssign HW 1 #8)*

   (b) Find all values of x in the interval $[0, 2\pi]$ that satisfy $\cot x \csc x = 4\cos x$. *(Similar to Written HW 1 #1)*

   (c) You see a bear cub in a tree on campus. Your distance from the tree is 100 ft. The angle between the ground and a straight line from your foot to the bear cub is $\pi/6$ radians. How high up is the bear cub? *(Similar to Written HW 1 #7a)*

   Solution:

   (a) Use a reference triangle. If the opposite side has length 1 and the adjacent side has length 10, then the hypotenuse has length $\sqrt{101}$. In Q3 $\cos\theta = \boxed{-10/\sqrt{101}}$.

   Alternate Solution:
   In Q3 $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{-1}{\sqrt{1+\tan^2\theta}} = \dfrac{-1}{\sqrt{101/100}} = \boxed{-\dfrac{10}{\sqrt{101}}}$.

   (b) $\cot x \csc x = 4\cos x \;\Rightarrow\; \dfrac{\cos x}{\sin^2 x} = 4\cos x$.

   Then $\cos x = 0 \;\Rightarrow\; x = \boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$ or $\sin x = \pm\dfrac{1}{2} \;\Rightarrow\; x = \boxed{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$.

   (c) Let h be the height of the tree. We are given that $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} = \frac{h}{100}$ so $h = \boxed{\frac{100}{\sqrt{3}}\text{ ft}}$.

2. **Core Section: Limits** (30 points)
   Evaluate the following limits or explain why they don't exist. Justify all answers.

   (a) $\displaystyle\lim_{x\to 0} \frac{x\cos x}{2\tan(2x)}$ *(Similar to WebAssign HW 5 #4)*

   (b) $\displaystyle\lim_{t\to 4} \frac{12-3t}{|t-4|}$ *(Similar to WebAssign HW 7 #8)*

   (c) $\displaystyle\lim_{x\to\infty} \frac{\sin^2 x}{x}$ *(Similar to Exam Review Sheet #11a)*

   (d) $\displaystyle\lim_{x\to 0} \frac{3-\sqrt{9-x^2}}{x^2}$ *(Similar to WebAssign HW 5 #2)*

   Solution:

(a) Use the theorem $\lim_{\theta\to 0}\dfrac{\sin\theta}{\theta} = \lim_{\theta\to 0}\dfrac{\theta}{\sin\theta} = 1.$

$$\lim_{x\to 0}\frac{x\cos x}{2\tan(2x)} = \lim_{x\to 0}\frac{x\cos x}{2}\cdot\frac{\cos(2x)}{\sin(2x)}\cdot\frac{2}{2} = \lim_{x\to 0}\frac{\cos x}{2}\cdot\frac{2x}{\sin(2x)}\cdot\frac{\cos(2x)}{2} = \frac{1}{2}\cdot 1\cdot\frac{1}{2} = \boxed{\frac{1}{4}}.$$

(b)

$$\lim_{t\to 4^+}\frac{12-3t}{|t-4|} = \lim_{t\to 4^+}\frac{3(4-t)}{t-4} = -3.$$
$$\lim_{t\to 4^-}\frac{12-3t}{|t-4|} = \lim_{t\to 4^-}\frac{3(4-t)}{4-t} = 3.$$

It follows that $\lim_{t\to 4}\dfrac{12-3t}{|t-4|}$ $\boxed{\text{does not exist}}$ because the right-hand and left-hand limits are not equal.

(c) Because $0 \le \sin^2 x \le 1 \;\Rightarrow\; 0 \le \dfrac{\sin^2 x}{x} \le \dfrac{1}{x}$ and $\lim_{x\to\infty}\dfrac{1}{x} = 0$, by the Squeeze Theorem the value of $\lim_{x\to\infty}\dfrac{\sin^2 x}{x}$ also is $\boxed{0}$.

(d) Multiply by the conjugate of the numerator.

$$\lim_{x\to 0}\frac{3-\sqrt{9-x^2}}{x^2}\cdot\frac{3+\sqrt{9-x^2}}{3+\sqrt{9-x^2}} = \lim_{x\to 0}\frac{9-\left(9-x^2\right)}{x^2\left(3+\sqrt{9-x^2}\right)} = \lim_{x\to 0}\frac{\cancel{x^2}}{\cancel{x^2}\left(3+\sqrt{9-x^2}\right)} = \boxed{\frac{1}{6}}.$$

3. (20 points) Let $h(x) = \dfrac{x^2-9}{2x^2+4x-6}$.

   (a) Find the domain of $h(x)$. Express your answer in interval notation. *(Similar to WebAssign HW 2 #5)*

   (b) Find and classify all discontinuities of $h(x)$. Justify using appropriate limits. *(Similar to Exam Review Sheet #14)*

   (c) Find the horizontal asymptotes, if any. Justify using appropriate limits. *(Similar to Written HW 3 #7)*

Solution:

(a) The function is undefined where the denominator $2x^2+4x-6 = 2(x+3)(x-1)$ equals 0 at $x = -3, 1$. The domain is $\boxed{(-\infty,-3)\cup(-3,1)\cup(1,\infty)}$.

(b) There is a $\boxed{\text{removable discontinuity}}$ at $\boxed{x=-3}$:

$$\lim_{x\to -3}\frac{x^2-9}{2x^2+4x-6} = \lim_{x\to -3}\frac{\cancel{(x+3)}(x-3)}{2\cancel{(x+3)}(x-1)} = \frac{3}{4}.$$

There is an $\boxed{\text{infinite discontinuity}}$ at $\boxed{x=1}$:

$$\lim_{x\to 1^+} \frac{x^2-9}{2x^2+4x-6} = \lim_{x\to 1^+} \frac{x-3}{2(x-1)} = -\infty$$

since the numerator approaches -2 and the denominator approaches 0 with positive values.

(c) There is a horizontal asymptote at $\boxed{y=1/2}$:

$$\lim_{x\to\infty} \frac{x^2-9}{2x^2+4x-6} = \lim_{x\to\infty} \frac{1-\frac{9}{x^2}}{2+\frac{4}{x}-\frac{6}{x^2}} = \frac{1}{2}.$$

$$\lim_{x\to-\infty} \frac{x^2-9}{2x^2+4x-6} = \lim_{x\to-\infty} \frac{1-\frac{9}{x^2}}{2+\frac{4}{x}-\frac{6}{x^2}} = \frac{1}{2}.$$

4. (10 points) Show that the equation $7\sqrt{x} = -3\sec x + 10$ has at least one real solution. Indicate the interval where a solution can be found. *(Similar to Written HW 3 #1)*

Solution: We use the fact that $\sqrt{\pi/3} > 1$.

Option 1: Use the Intermediate Value Theorem to show that $f(x) = 7\sqrt{x} + 3\sec(x) - 10 = 0$ is equal to zero.

$$f(0) = 0 + 3(1) - 10 < 0.$$
$$f(\pi/3) = 7\sqrt{\pi/3} + 3(2) - 10 > 0.$$

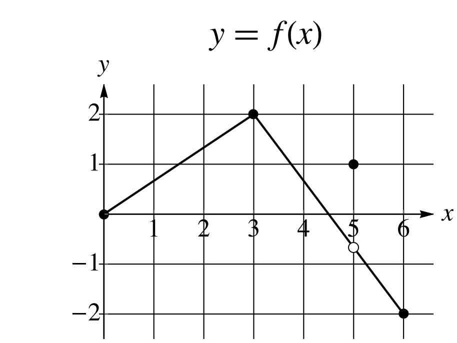
Since $f(0) < 0$ and $f(\pi/3) > 0$ and f is continuous on $[0, \pi/3]$, the Intermediate Value Theorem guarantees that $f(x) = 0$ has a solution in the interval $\boxed{(0, \pi/3)}$.

Option 2: Use the Intermediate Value Theorem to show that $g(x) = 7\sqrt{x} + 3\sec(x)$ is equal to 10.

$$g(0) = 0 + 3(1) < 10.$$
$$g(\pi/3) = 7\sqrt{\pi/3} + 3(2) > 10.$$

Since $g(0) < 10$ and $g(\pi/3) > 10$ and g is continuous on $[0, \pi/3]$, the Intermediate Value Theorem guarantees that $g(x) = 10$ has a solution in the interval $\boxed{(0, \pi/3)}$.

5. **Solution:**



(20 points) Shown above is a graph of $y = f(x)$ which consists of two line segments with a single removable discontinuity.

(a) Find a formula for $f(x)$. *(Similar to Written HW 1 #4)*

(b) Can the exact value of $\lim_{x \to 5} |f(x)|$ be determined? If so, find the value of the limit. If not, explain why not. *(Similar to WebAssign HW 5 #3)*

(c) Sketch a graph of $y = |f(x) - 1|$. Label the intercepts, if any. *(Similar to WebAssign HW 2 #11)*

(d) Suppose we use the precise definition of a limit to verify the value of $\lim_{x \to 1} f(x)$ and we find that if $\frac{3}{4} < x < \frac{5}{4}$ then $\frac{1}{2} < f(x) < \frac{5}{6}$. What are the corresponding values of ϵ and δ? *(Similar to Written HW 2 #3)*

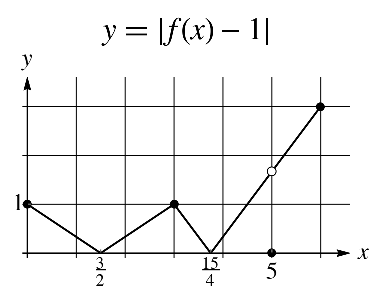
Precise Definition of a Limit: The limit of $f(x)$ as x approaches a is L if for every number $\epsilon > 0$ there is a corresponding number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

Solution:

(a) $$f(x) = \begin{cases} \frac{2}{3}x, & 0 \le x < 3 \\ -\frac{4}{3}x + 6, & 3 \le x < 5 \\ 1, & x = 5 \\ -\frac{4}{3}x + 6, & 5 < x \le 6 \end{cases}$$

(b) $\boxed{\text{Yes.}}$ $\lim_{x \to 5} |f(x)| = \lim_{x \to 5} \left|-\frac{4}{3}x + 6\right| = \left|-\frac{20}{3} + 6\right| = \boxed{\frac{2}{3}}$.

(c)



(d) Note that $f(1) = \frac{2}{3}$. The intervals correspond to $|x - 1| < \frac{1}{4}$ and $\left|f(x) - \frac{2}{3}\right| < \frac{1}{6}$ so the values are $\boxed{\epsilon = \frac{1}{6} \text{ and } \delta = \frac{1}{4}}$.