
On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- **Show all work and simplify your answers!** Name any theorem that you use. Limit problems should not be evaluated using L'Hopital's Rule. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

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1. **Core Section: Trigonometry** (20 points)
   (a) Given $\tan\theta = 1/10$ and $\pi < \theta < 3\pi/2$, what is $\cos\theta$?
   (b) Find all values of x in the interval $[0, 2\pi]$ that satisfy $\cot x \csc x = 4\cos x$.
   (c) You see a bear cub in a tree on campus. Your distance from the tree is 100 ft. The angle between the ground and a straight line from your foot to the bear cub is $\pi/6$ radians. How high up is the bear cub?

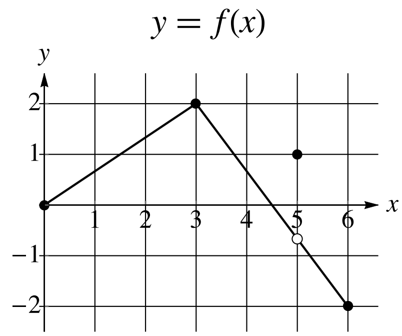
2. **Core Section: Limits** (30 points)
   Evaluate the following limits or explain why they don't exist. Justify all answers.
   (a) $\displaystyle\lim_{x\to 0} \frac{x\cos x}{2\tan(2x)}$
   (b) $\displaystyle\lim_{t\to 4} \frac{12-3t}{|t-4|}$
   (c) $\displaystyle\lim_{x\to\infty} \frac{\sin^2 x}{x}$
   (d) $\displaystyle\lim_{x\to 0} \frac{3-\sqrt{9-x^2}}{x^2}$

3. (20 points) Let $h(x) = \dfrac{x^2-9}{2x^2+4x-6}$.
   (a) Find the domain of $h(x)$. Express your answer in interval notation.
   (b) Find and classify all discontinuities of $h(x)$. Justify using appropriate limits.
   (c) Find the horizontal asymptotes, if any. Justify using appropriate limits.

4. (10 points) Show that the equation $7\sqrt{x} = -3\sec x + 10$ has at least one real solution. Indicate the interval where a solution can be found.

TURN OVER—More problems on the back!

5.



(20 points) Shown above is a graph of $y = f(x)$ which consists of two line segments with a single removable discontinuity.

(a) Find a formula for $f(x)$.

(b) Can the <u>exact</u> value of $\lim_{x\to 5} |f(x)|$ be determined? If so, find the value of the limit. If not, explain why not.

(c) Sketch a graph of $y = |f(x) - 1|$. Label the intercepts, if any.

(d) Suppose we use the precise definition of a limit to verify the value of $\lim_{x\to 1} f(x)$ and we find that if $\frac{3}{4} < x < \frac{5}{4}$ then $\frac{1}{2} < f(x) < \frac{5}{6}$. What are the corresponding values of ϵ and δ?

Precise Definition of a Limit: The limit of $f(x)$ as x approaches a is L if for every number $\epsilon > 0$ there is a corresponding number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.