On the front of your bluebook, please write: a grading key, your name, lecture number, and instructor name. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Make sure all of your work is in your bluebook. Nothing on this exam sheet will be graded. Please begin each problem on a new page.
- Show all work and simplify your answers! Name any theorem that you use. Limit problems should not be evaluated using L’Hôpital’s Rule. Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

1. **Core Section: Trigonometry** (20 points)
   
   (a) Given \( \tan \theta = \frac{1}{10} \) and \( \pi < \theta < \frac{3\pi}{2} \), what is \( \cos \theta \)?
   
   (b) Find all values of \( x \) in the interval \([0, 2\pi]\) that satisfy \( \cot x \csc x = 4 \cos x \).
   
   (c) You see a bear cub in a tree on campus. Your distance from the tree is 100 ft. The angle between the ground and a straight line from your foot to the bear cub is \( \frac{\pi}{6} \) radians. How high up is the bear cub?

2. **Core Section: Limits** (30 points)
   
   Evaluate the following limits or explain why they don’t exist. Justify all answers.
   
   (a) \( \lim_{x \to 0} \frac{x \cos x}{2 \tan(2x)} \)
   
   (b) \( \lim_{t \to 4} \frac{12 - 3t}{|t - 4|} \)
   
   (c) \( \lim_{x \to \infty} \frac{\sin^2 x}{x} \)
   
   (d) \( \lim_{x \to 0} \frac{3 - \sqrt{9 - x^2}}{x^2} \)

3. (20 points) Let \( h(x) = \frac{x^2 - 9}{2x^2 + 4x - 6} \).
   
   (a) Find the domain of \( h(x) \). Express your answer in interval notation.
   
   (b) Find and classify all discontinuities of \( h(x) \). Justify using appropriate limits.
   
   (c) Find the horizontal asymptotes, if any. Justify using appropriate limits.

4. (10 points) Show that the equation \( 7\sqrt{x} = -3 \sec x + 10 \) has at least one real solution. Indicate the interval where a solution can be found.
(20 points) Shown above is a graph of \( y = f(x) \) which consists of two line segments with a single removable discontinuity.

(a) Find a formula for \( f(x) \).

(b) Can the exact value of \( \lim_{x \to 5} |f(x)| \) be determined? If so, find the value of the limit. If not, explain why not.

(c) Sketch a graph of \( y = |f(x) - 1| \). Label the intercepts, if any.

(d) Suppose we use the precise definition of a limit to verify the value of \( \lim_{x \to 1} f(x) \) and we find that if \( \frac{3}{4} < x < \frac{5}{4} \) then \( \frac{1}{2} < f(x) < \frac{5}{6} \). What are the corresponding values of \( \epsilon \) and \( \delta \)?

Precise Definition of a Limit: The limit of \( f(x) \) as \( x \) approaches \( a \) is \( L \) if for every number \( \epsilon > 0 \) there is a corresponding number \( \delta > 0 \) such that if \( 0 < |x - a| < \delta \) then \( |f(x) - L| < \epsilon \).