APPM 1345
Final Exam
Spring 2024

Name	
Instructor	**Lecture Section**

This exam is worth 150 points and has **8 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, you may ask one of your proctors for a piece of scratch paper. Do NOT use any paper that you have brought with you.

Show all work and *simplify* your answers. Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

End of Exam Check List

1. If you finish the exam before 12:45 PM:
   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.
2. If you finish the exam after 12:45 PM:
   - Please wait in your seat until 1:00 PM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.

Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad \cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C \qquad \int \frac{1}{1+x^2}\,dx = \arctan(x) + C$$

1. (28 points) The following problems are not related.

   (a) Find the general antiderivative of $g(x) = \dfrac{e^{\sqrt{x}}}{\sqrt{x}}$.

   (b) Use logarithmic differentiation to find the derivative of $y = (x^4 + 1)^x$. *You do not need to simplify your answer.*

   (c) Find the derivative of $f(x) = \displaystyle\int_0^{\cos(x)} \sqrt{1 + t^3}\, dt$.

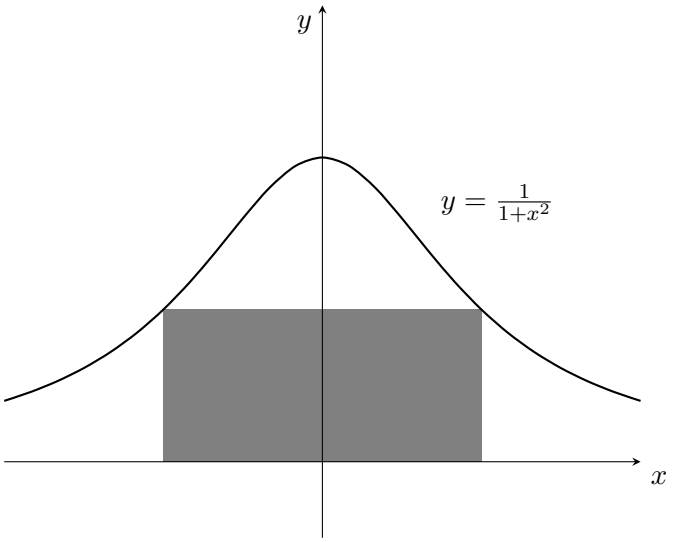
2. (26 points) The following problems are not related:

   (a) Find the derivative of $f(x) = \ln\left(\tan^{-1}(x)\right)$.

   (b) Evaluate the definite integral $\displaystyle\int_0^{\ln(3)} \sinh(x)\cosh(x)\,dx$, and fully simplify your answer.

   (c) Determine the value of the limit $\displaystyle\lim_{x\to 0^+} x^2\ln(x^2)$.

3. (16 points) Find the area of the largest rectangle which is symmetric around the y-axis, bounded below by the x-axis, and which has two corners touching the graph of $f(x) = \dfrac{1}{1+x^2}$. *Fully justify your answer by using an appropriate test.*



4. (18 points) A bug flying in a straight line starts decelerating at time $t = 0$ at a constant rate of 1 ft/s^2 for 5 seconds. Answer the following questions about the bug over the time interval $0 \leq t \leq 5$.

   (a) Find the bug's velocity as a function of time, given that its velocity at $t = 0$ is 2 ft/s.

   (b) What is the bug's displacement over the time interval $0 \leq t \leq 5$?

   (c) The bug changes direction at least once during the 5 seconds. What is the total distance the bug travels over the time interval $0 \leq t \leq 5$?

5. (12 points) For what value of a is the following function continuous?

$$f(x) = \begin{cases} 2x^2 - x + a, & x \leq 0 \\ \dfrac{x}{2\sin(x)}, & x > 0 \end{cases}$$

Justify your answer with appropriate computations.

6. (18 points) Consider the function

$$g(x) = \arctan(x) + \frac{1}{x^2 - 4}$$

(a) Find the domain of the function, and give your answer in interval notation.

(b) Find all horizontal asymptotes of $g(x)$, and justify your answer with limits.

7. (16 points) The half-life of the chemical element cobalt-56 is approximately 77 days. Suppose we have a 10 milligram sample of cobalt-56..

   (a) Find a formula for the mass of cobalt-56 remaining after t days.

   (b) How long will it take for only 1 milligram of cobalt-56 to remain in the sample? *It is OK for your answer to have a logarithm in it.*

8. (16 points) For each of the following questions, give a short justification for your answer.

(a) If $f(x)$ is an odd function and $\int_{-3}^{0} f(x)\,dx = \pi + 1$, find $\int_{-3}^{3} f(x)\,dx$.

(b) Find the absolute minimum of the function $f(x) = x \cdot 2^x$, if it exists.

(c) Evaluate the limit $\lim_{h \to 0} \dfrac{\arctan(3x + 3h) - \arctan(3x)}{h}$.

(d) Suppose that $f(x)$ is differentiable everywhere, with $f(-1) = 1$ and $f(1) = 3$. Is there some value c such that $f'(c) = 1$?

END OF TEST

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.