

1. (44 pts) Evaluate the following expressions. Fully simplify your answers.

(a) $\lim_{x \rightarrow 0} \frac{\sin(x/2)}{e^{2x} - 1}$

(b) $\frac{d}{dx} \int_0^{x^2} e^t \cos t \, dt$

(c) $\frac{d}{dx} ((\sec x)^x)$

(d) $\int \frac{\tan \theta}{\cos^2 \theta} \, d\theta$

(e) $\sum_{i=1}^6 \ln \left(\frac{i+3}{i+2} \right)$ (Write your answer as a single log expression.)

2. (12 pts) A particle is moving along a straight line. The position function of the particle in meters after t seconds is given by

$$s(t) = \frac{t^3}{3} - \frac{5t^2}{2} + 6t, \quad 1 \leq t \leq 5.$$

- (a) Find the particle's instantaneous velocity $v(t)$ at $t = 4$ seconds.
(b) What is the average value of the acceleration $a(t)$ on the interval $[1, 5]$?

3. (28 pts) Let $g(x) = \frac{x}{1+4x}$.

- (a) What is the domain of the function? Express your answer in interval notation.
(b) Find $g'(x)$ and simplify.
(c) Find the inverse function $g^{-1}(x)$.

(d) Evaluate $\int_1^6 \frac{1}{1+4x} \, dx$.

4. (12 pts) The Shiveluch volcano on the Kamchatka peninsula is currently erupting and forming a lava dome in the shape of a hemisphere.

- (a) When the radius of the dome is 10 meters, it is increasing at a rate of 2 meters/hour. How fast is the volume of the dome changing? (The volume of a hemisphere is $V = \frac{2}{3}\pi r^3$.)
(b) Assume that the volume's rate of change remains constant. Find the radius when it is increasing at a rate of 1 meter/hour.

5. (28 pts) The following two problems are not related.

(a) Let $f(x) = \frac{\sin^{-1}(x)}{x}$.

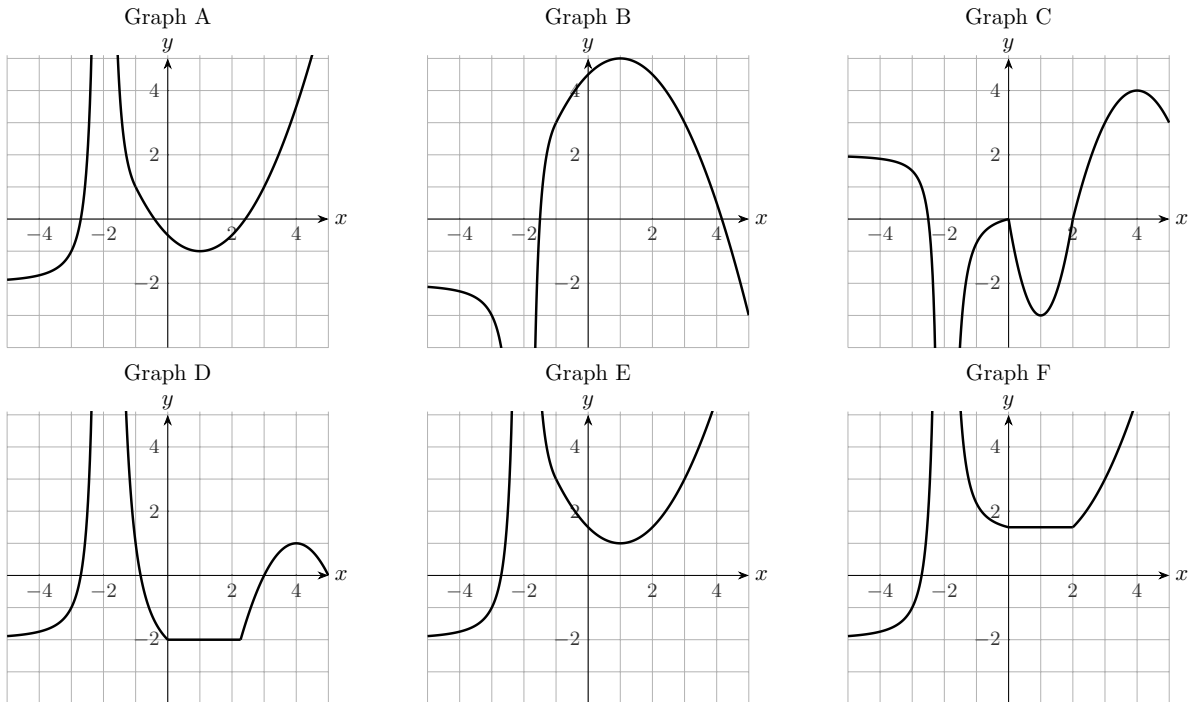
- i. Find the values of $f(-1)$ and $f(1)$.
- ii. Does $f(x)$ have any vertical asymptotes? If so, find them. Justify your answer using limits.

(b) Let $h(x) = \sinh(\ln x)$.

- i. Find $h'(x)$.
- ii. Find an equation of the line tangent to the curve $y = h(x)$ at $x = 3$. Write your fully simplified answer in slope-intercept form with no hyperbolic functions.

6. (12 pts) A sample of the radioactive element Unobtainium decayed by 10% in one day. In hours, how long did it take for the sample to decay by 3%?

7. (14 pts) Consider the six graphs A, B, C, D, E, and F shown below. No justification is necessary for the following questions.



(a) Which of the six graphs satisfies all of the following conditions?

- $\lim_{x \rightarrow -\infty} f(x) = -2$
- $\lim_{x \rightarrow -2} f(x) = \infty$
- $f'(1) = 0$ and $f''(1) > 0$
- the line $y = 2x - 3$ is tangent to f at $x = 3$

(b) Which of the six graphs is the derivative graph of the function $y = r(x)$ shown below?

