APPM 1345

Exam 2

Spring 2023

Name	
Instructor Richard McNamara	**Section** 150

This exam is worth 100 points and has **4 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

Show all work and simplify your answers. Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are <u>not permitted</u>.

End-of-Exam Checklist

1. If you finish the exam before 7:45 PM:

   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors.

2. If you finish the exam after 7:45 PM:

   - Please wait in your seat until 8:00 PM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors.

Formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

1. (21 pts) The following are related to the definite integal $A = \int_0^2 x^2 dx$.

    (a) Approximate the value of A using a Riemann sum with a regular partition of $n = 4$ subintervals and right endpoints (that is, determine the value of R_4). Fully simplify your answer.

    (b) Find an expression for R_n. Express your answer using sigma notation and fully simplify your result.

    (c) Determine the exact value of A by taking the appropriate limit of your result from part (b). Simplify your answer fully. (No credit will be earned by using the Evaluation Theorem here.)

2. (23 pts) Parts (a) and (b) are unrelated.

(a) Evaluate $\int_{-3}^{4} f(x)dx$, where f is defined as:

$$f(x) = \begin{cases} 2x + 4 & , \quad -3 \leq x \leq 0 \\ \sqrt{16 - x^2} & , \quad 0 < x \leq 4 \end{cases}$$

Fully simplify your answer. (*Hint:* Consider the relationship between definite integrals and areas.)

(b) Let g be a continuous function on $[-3, 3]$ with $\displaystyle\int_{-1}^{0} g(x)dx = 4$ and $\displaystyle\int_{0}^{3} g(x)dx = 11$.

Find the value of $\displaystyle\int_{-3}^{-1} g(x)dx$ in each of the following cases:

i. g is an even function

ii. g is an odd function

3. (24 pts) Parts (a) and (b) are unrelated.

(a) Suppose the velocity function of a car is $v(t) = 50 + 2t^3$ miles per hour between $t = 0$ hours and $t = 2$ hours. Determine the car's average velocity during that time period. Fully simplify your answer and include the correct unit of measurement.

(b) Evaluate the following derivatives. Fully simplify your answers.

i. $\dfrac{d}{dx} \displaystyle\int_x^9 \sqrt{1 + t^4}\, dt$

ii. $\dfrac{d}{dx} \displaystyle\int_{2x}^{x^2} t \sin t\, dt$

4. (32 pts) Evaluate the following integrals. Fully simplify your answers.

(a) $\displaystyle\int \frac{\sin x}{\cos^4 x}\, dx$ (Express your answer in terms of x)

(b) $\displaystyle\int_0^2 9x^2\sqrt{2x^3 + 1}\, dx$

(c) $\int x(x-3)^{1/5}\,dx$ (Express your answer in terms of x)

ADDITIONAL BLANK SPACE
If you write a solution here, please clearly indicate the problem number.