INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your full name**, (2) **1345/Exam 3**, (3) **lecture number/instructor name** and (4) **SPRING 2022** on the front of your bluebook. Do all problems. **Start each problem on a new page. Box your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work.**

---

1. (24pts) The following *parts* of this **problem** are not related.

   (a)(12pts) Suppose the function $g(x) = \dfrac{x+2}{x-3}$ is *one-to-one*, find the inverse $g^{-1}(x)$. Show all work.

   (b)(12pts) If f is a one-to-one function with $f(0) = 7$ and $f'(0) = 3$, find $\dfrac{d}{dx} f^{-1}(7)$ given $[f^{-1}(a)]' = [f'(f^{-1}(a))]^{-1}$.

---

2. (28pts) Start this **problem** on a new page. The following *parts* are not related.

   (a)(12pts) A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420. Write down the solution of the differential equation $\dfrac{dy}{dt} = ky$, $y(0) = y_0$ (no justification necessary for the solution of the DE) and then find the *relative growth rate*, k, of the bacteria population based on the given information.

   (b)(12pts) Use the *Product Rule* to find the derivative of the function $f(x) = \sin(x)\ln(x^2+1)$.

   (c)(4pts) *Multiple Choice:* If we use the following definition of the derivative: $f'(a) = \lim\limits_{x\to a} \dfrac{f(x)-f(a)}{x-a}$ to evaluate the limit $\lim\limits_{x\to 0} \dfrac{\ln(1+2x)}{x}$ then which choice below do we get?
   (**No justification necessary, choose only one answer** – *copy down the entire answer in your bluebook.*)

   (A) 0 (B) $\dfrac{0}{0}$ (C) $\dfrac{1}{2}$ (D) 1 (E) 2

PROBLEMS #3 & #4 ON THE NEXT PAGE

3. (24pts) Start this **problem** on a new page. The following *parts* are not related.

(a)(12pts) Use the *Quotient Rule* to find the $f'(x)$ if $f(x) = \dfrac{e^x}{1+e^x}$. Simplify your answer.

(b)(12pts) Use *logarithmic differentiation* to find the derivative of: $\dfrac{(x+1)^4}{(x-3)^8}$.

---

4. (24pts) Start this **problem** on a new page. The following *parts* are not related.

(a)(10pts) Use u-substitution to find the antiderivative: $\displaystyle\int \frac{\ln(1+2x)}{2x+1}\,dx$.

(b)(10pts) Evaluate the definite integral: $\displaystyle\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{e^{1/x}}{x^2}\,dx$.

(c)(4pts) *Multiple Choice:* The *horizontal asymptotes* of the function $f(x) = \dfrac{e^{2x}-e^x}{e^{2x}+1}$ are given by which choice below? (**No justification necessary, choose only one answer** – *copy down the entire answer in your bluebook.*)

(A) $y = 0, 1$ (B) $y = \pm 1$ (C) $y = 0, \dfrac{1}{2}$ (D) $y = \dfrac{1}{e^2}, 1$ (E) $y = \dfrac{1}{e}, 1$

— END —