INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write (1) your full name, (2) 1345/Exam 2, (3) lecture number/instructor name and (4) SPRING 2022 on the front of your bluebook. Do all problems. Start each problem on a new page. Box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. Justify your answers, show all work.

1. (24pts) The following problems are not related.

   (a) (12pts) Find the value of the sum \( \sum_{i=1}^{n} \frac{1}{n} \left[ i + \frac{i^2}{n^2} \right] \) in terms of \( n \). (Do not take any limits.) You may or may not find the following formulas useful:
   \[
   \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \text{and} \quad \sum_{i=1}^{n} i^3 = \left[ \frac{n(n+1)}{2} \right]^2.
   \]

   (b) (12pts) Use the Fundamental Theorem of Calculus to evaluate the integral: \( \int_{0}^{2} (2 - x^2) \, dx \)

2. (28pts) Start this problem on a new page. The following problems are not related.

   (a) (12pts) Approximate the area under the curve \( y = x^2 + 2x + 4 \) from \( x = 0 \) to \( x = 6 \) with a Riemann sum using \( n = 3 \) subintervals of equal width and left endpoints (that is, find the approximation \( L_3 \)).

   (b) (12pts) Write the expression \( \int_{-2}^{2} f(t) \, dt - \int_{-2}^{-1} f(x) \, dx \) as a single integral in the form \( \int_{b}^{a} f(x) \, dx \).

   (c) (4pts) (Multiple Choice) Using right endpoints (\( R_n \)) and subintervals of equal width, which limit below is equal to the definite integral \( \int_{3}^{1} \frac{x}{x^2 + 4} \, dx \)? (No justification necessary - Choose only one answer, copy down the entire answer.)

      (A) \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2i}{(2i/n)^2 + 4} \cdot \frac{2}{n} \)
      (B) \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{1 + 2i/n}{(1 + 2i/n)^2 + 4} \cdot \frac{2}{n} \)
      (C) \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{1 + 2(i - 1)/n}{(1 + 2(i - 1)/n)^2 + 4} \cdot \frac{2}{n} \)
      (D) \( \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2i}{(2i/n)^2 + 4} \)

PROBLEMS #3 & #4 ON THE OTHER SIDE
3. (24pts) Start this problem on a **new** page. The following problems are not related.

(a) (12pts) Evaluate the definite integral \( \int_{0}^{3} |x - 2| \, dx \).

(b) (12pts) Use a \( u \)-substitution to evaluate the indefinite integral \( \int x\sqrt{x - 1} \, dx \). Show all work.

---

4. (24pts) Start this problem on a **new** page. The following problems are not related.

(a) (10pts) Evaluate the definite integral: \( \int_{0}^{\pi/2} \sin^2(x) \cos(x) \, dx \)

(b) (10pts) If \( f(x) = \int_{4}^{x^2} \frac{t - 1}{t^2 + 1} \, dt \), use the Fundamental Theorem of Calculus to find \( f'(2) \). Simplify your answer.

(c) (4pts) Suppose we have a rectangle of width \( w = 4 \), what should the height, \( h \), of the rectangle be so that the area of the rectangle and the area bounded by the curve \( f(x) = \sqrt{x} \), for \( 0 \leq x \leq 4 \), and the \( x \)-axis are the same?  

(No justification necessary - Choose only **one** answer, copy down the **entire** answer)

- (A) \( h = \frac{1}{2} \)
- (B) \( h = \frac{4}{3} \)
- (C) \( h = \frac{1}{4} \)
- (D) \( h = \frac{2}{3} \)
- (E) None of these

---

— END —