INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your full name**, (2) **1345/Exam 2**, (3) **lecture number/instructor name** and (4) **SPRING 2022** on the front of your bluebook. Do all problems. **Start each problem on a new page.** **Box** **your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work.**

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1. (24pts) The following problems are not related.

   (a)(12pts) Find the value of the sum $\displaystyle\sum_{i=1}^{n} \frac{1}{n}\left[\frac{i}{n} + \frac{i^2}{n^2}\right]$ in terms of n. (Do **not** take any limits.). You may or may not find the following formulas useful:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \text{and} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

   (b)(12pts) Use the Fundamental Theorem of Calculus to evaluate the integral: $\displaystyle\int_0^2 (2 - x^2)\, dx$

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2. (28pts) Start this problem on a **new** page. The following problems are not related.

   (a)(12pts) Approximate the area under the curve $y = x^2 + 2x + 4$ from $x = 0$ to $x = 6$ with a *Riemann sum* using $n=3$ subintervals of equal width and left endpoints (that is, find the approximation L_3).

   (b)(12pts) Write the expression $\displaystyle\int_2^5 f(x)\, dx + \int_{-2}^{2} f(t)\, dt - \int_{-2}^{-1} f(x)\, dx$ as a single integral in the form $\displaystyle\int_a^b f(x)\, dx$.

   (c)(4pts) *(Multiple Choice)* Using right endpoints (R_n) and subintervals of equal width, which limit below is equal to the definite integral $\displaystyle\int_1^3 \frac{x}{x^2+4}\, dx$? (**No justification necessary** - *Choose only one answer, copy down the entire answer.*)

   (A) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i/n}{(2i/n)^2 + 4} \cdot \frac{2}{n}$ (B) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1 + 2i/n}{(1 + 2i/n)^2 + 4} \cdot \frac{2}{n}$ (C) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1 + 2(i-1)/n}{(1 + 2(i-1)/n)^2 + 4} \cdot \frac{2i}{n}$ (D) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i/n}{(2i/n)^2 + 4}$

PROBLEMS #3 & #4 ON THE OTHER SIDE

3. (24pts) Start this problem on a **new** page. The following problems are not related.

(a)(12pts) Evaluate the definite integral $\displaystyle\int_0^3 |x-2|\,dx$.

(b)(12pts) Use a u-substitution to evaluate the indefinite integral $\displaystyle\int x\sqrt{x-1}dx$. Show all work.

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4. (24pts) Start this problem on a **new** page. The following problems are not related.

(a)(10pts) Evaluate the definite integral: $\displaystyle\int_0^{\pi/2} \sin^2(x)\cos(x)\,dx$

(b)(10pts) If $f(x) = \displaystyle\int_4^{x^2} \frac{t-1}{t^2+1}\,dt$, use the Fundamental Theorem of Calculus to find $f'(2)$. Simplify your answer.

(c)(4pts) Suppose we have a rectangle of width $w = 4$, what should the height, h, of the rectangle be so that the area of the rectangle and the area bounded by the curve $f(x) = \sqrt{x}$, for $0 \le x \le 4$, and the x-axis are the same?
(**No justification necessary** - *Choose only one answer, copy down the entire answer*)

(A) $h = \frac{1}{2}$ (B) $h = \frac{4}{3}$ (C) $h = \frac{1}{4}$ (D) $h = \frac{2}{3}$ (E) None of these

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