INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your full name**, (2) **1345/Exam 1**, (3) **lecture number/instructor name** and (4) **SPRING 2022** on the front of your bluebook. Do all problems. **Start each problem on a new page.** **Box** **your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work.**

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1. (24pts) The following problems are not related.

   (a)(12pts) Find all the intervals on which $g(x) = 3x^5 - 5x^3$ is *increasing* or *decreasing*. **Give your answer in interval notation.** Show all work.

   (b)(12pts) Find and classify any critical point(s) of $f(x) = x + 2\cos(x),\ 0 \le x \le \dfrac{\pi}{2}$. Classify the critical point using the **2nd Derivative Test**. (You do not need to find the y-value of any critical point.)

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2. (28pts) Start this problem on a **new** page. The following problems are not related.

   (a)(12pts) *Consider the following problem:* The monthly production of a light bulb company is $P = 4xy$ (in millions) where x is the cost of equipment and y is the cost of labor (in millions of dollars). The company needs to produce $P = 1$ million units, which values of x and y will minimize the cost $C = x + y$? **Answer the following questions:** *(i)*(4pts) Is this is a *minimization* or *maximization* problem? Write down a function in terms of the two variables x and y that you would minimize (or maximize). *(ii)*(4pts) Use the given information to write an equation that relates the variables x and y. *(iii)*(4pts) Now using optimization find the value of x and y that satisfy this problem. Justify your answer by classifying your critical point(s) using either the 1st or 2nd Derivative Test.

   (b)(12pts) Suppose we want to approximate the x-intercept of $f(x) = 3x^2 - 2$ using Newton's Method. What would the formula for x_{n+1} be? (To get full points for this question you must provide the explicit formula for x_{n+1} in terms of x_n, the generic formula for Newton's Method is not sufficient. You do *not* need to approximate the solution. **Simplify your answer.**)

   (c)(4pts) *Multiple Choice:* If $F(x) = \dfrac{x}{x^2+1}$ is an antiderivative of $f(x)$ then $f(x)$ is equal to which choice below? (**No justification necessary** - *Choose only one answer, copy down the entire answer.*)

   (A) $f(x) = \dfrac{x^2/2}{x^3/3 + 3x}$ (B) $f(x) = \dfrac{1-x^2}{x^4+2x^2+1}$ (C) $f(x) = \dfrac{1}{2x}$ (D) $f(x) = \dfrac{1-x^2}{(x+1)^2}$ (E) None of these

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PROBLEMS #3 & #4 ON THE OTHER SIDE

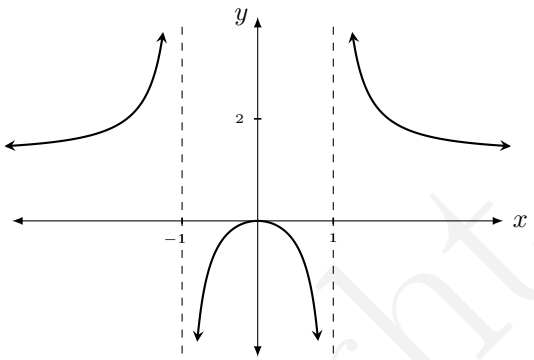
3. (24pts) Start this problem on a **new** page. The following parts of this problem are not related.

(a)(10pts) Find *any* function $F(x)$ such that $F'(x) = \dfrac{1+x^{5/2}}{x^{1/2}}$.

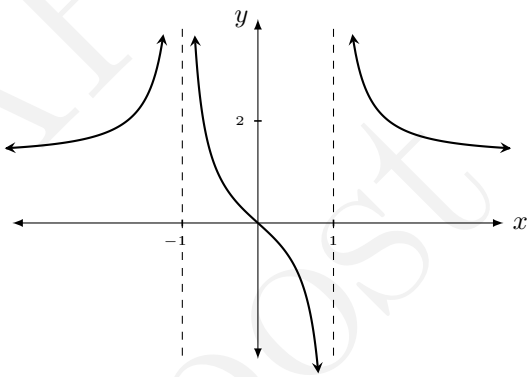
(b)(10pts) Find all *inflection points* of $g(x) = \dfrac{x^5}{20} - \dfrac{x^4}{6}$. Show all work and justify. (You do not need to find the y-value of any inflection point.)

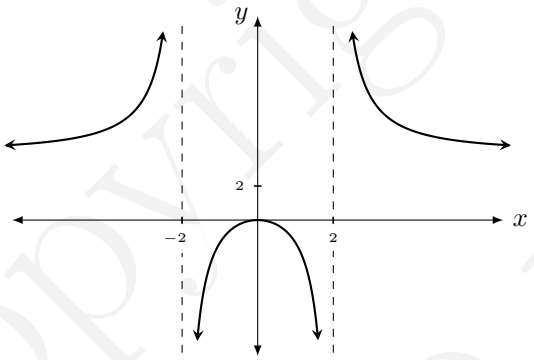
(c)(4pts) *Multiple Choice:* Which graph below *best* matches the graph of the function $f(x) = \dfrac{3x^2}{x^2-1}$?
(**No justification necessary** - *Choose only <u>one</u> answer, clearly indicate your answer otherwise points will be deducted.*)

Graph (A)

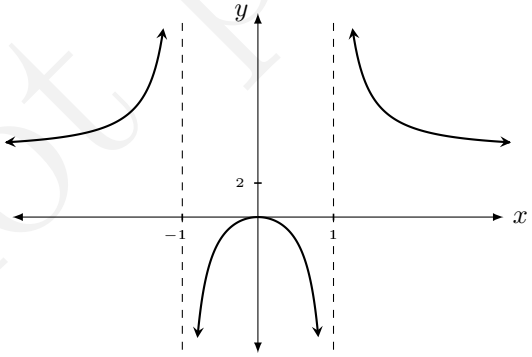


Graph (B)





Graph (C)



Graph (D)

4. (24pts) Start this problem on a **new** page. The following problems are not related.

(a)(12pts) Find the most general antiderivative of $f(t) = 2\sec(t)\tan(t) + \dfrac{1}{2t^2}$. Show all work.

(b)(12pts) Suppose the acceleration of an object at any time t is given by $a(t) = 3t^2 - 4t$ m/s^2, $t \geq 0$. Find the position, $s(t)$, at any time t if $v(1) = 1$ m/s and $s(0) = 2$. Show all work.

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