

INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your full name**, (2) **course number**, (3) **instructor name** and (4) **SPRING 2021** on the first page of your test. Do all problems. **Start each problem on a new page.** **Box** your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work.** You must start uploading your test to Gradescope by 1:01pm. Write the **honor code statement** given in the box below on the **first page** of your test and **sign** and **date it**.

On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this work. Signature: _____ Date: _____

1. (34pts) The following parts of this problems are not related.

(a)(17pts) Use the Quotient Rule to find the derivative of $f(x) = \frac{5x - 1}{\ln(x)}$. Simplify your answer.

(b)(17pts)(i)(5pts) Consider the limit $\lim_{x \rightarrow \infty} (5x)^{1/x}$. What type of indeterminate form is this limit?

(ii)(12pts) Evaluate the limit $\lim_{x \rightarrow \infty} (5x)^{1/x}$. Show all work.

2. (35pts) The following parts of this problems are not related. **Start this problem on a new page.**

(a)(16pts) Find the linearization of $f(x) = \arctan(x)$ at the point $a = 1$. Show all work.

(b)(16pts) Find the antiderivative: $\int \frac{3}{\sqrt{1 - 9x^2}} dx$. Show all work.

(c)(3pts) For which choice of the number c will the function $f(x) = \begin{cases} cx - \frac{1}{5}, & \text{if } x < 2, \\ \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6}, & \text{if } x \geq 2, \end{cases}$ be continuous?

(No justification necessary - Choose only one answer, copy down the entire answer.)

(A) $c=2$ (B) $c=-\frac{1}{2}$ (C) $c=-\frac{4}{3}$ (D) $c=5$ (E) None of these

3. (34pts) The following parts of this problems are not related. **Start this problem on a new page.**

(a)(17pts) Use *logarithmic differentiation* to find $\frac{dy}{dx}$ if $y = x^{\sin(x)}$. Your final answer should be in terms of the x -variable.

(b)(17pts) Given that the graph of $f(x) = 1 + 2x + \sinh^3(x)$ is one-to-one, find $(f^{-1})'(1)$. Justify.

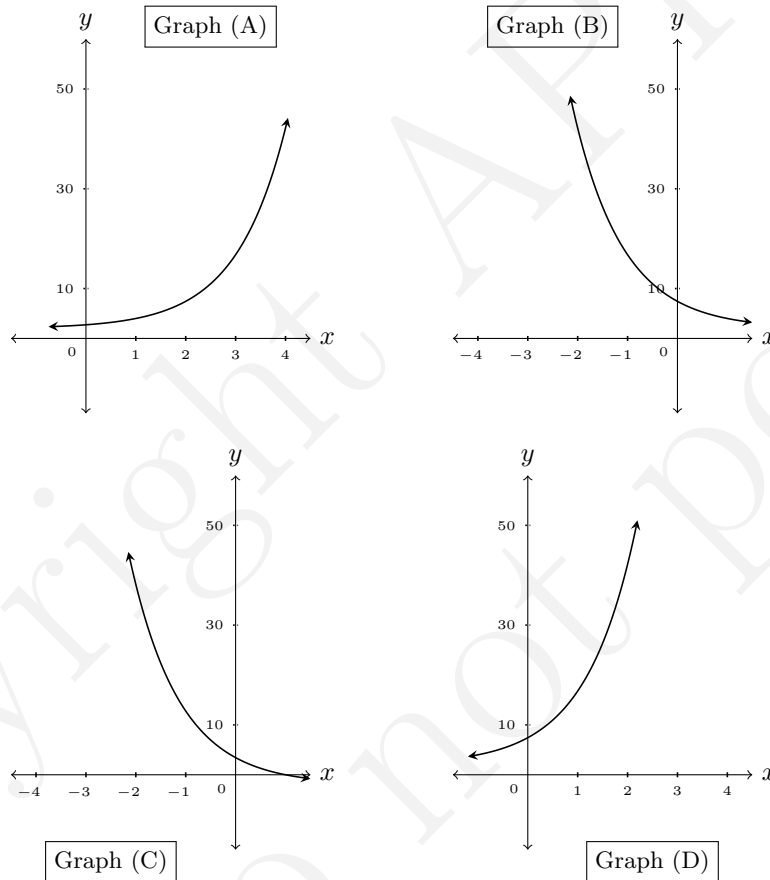
PROBLEMS #4 & #5 ON THE OTHER SIDE

4. (35pts) The following parts of this problems are not related. **Start this problem on a new page.**

(a)(16pts) Find the derivative of $g(x) = \int_e^{e^{2x}} \ln(t) dt$. Show all work and simplify your answer.

(b)(16pts) Evaluate the definite integral: $\int_0^{\sqrt{\pi}} x \cos(x^2) dx$. Show all work.

(c)(3pts) Which graph below best matches the graph of the function $f(x) = \frac{e}{e^x} + 1$? **Choose only one answer.** *No justification necessary, clearly indicate your answer otherwise points will be deducted.*



5. (12pts) Answer either **ALWAYS TRUE** or **FALSE**. You do **NOT** need to justify your answer. (*Don't just write down "A.T." or "F", completely write out the words "ALWAYS TRUE" or "FALSE" depending on your answer.*)

(a)(3pts) The sum $4 + 6 + 8 + 10 + 12$ can be written as $\sum_{i=11}^{15} [2i - 18]$.

(b)(3pts) We can show that $\lim_{h \rightarrow 0} \frac{5^{2+h} - 25}{h} = 25 \ln(5)$.

(c)(3pts) If, for an object moving in a straight line, we have $a(t) = 10 \frac{m}{s^2}$, $v(0) = 5 \frac{m}{s}$ and $s(0) = 0$, then $s(2) = 40m$.

(d)(3pts) Note that $\int_{-1}^1 |2x| dx = |x^2| \Big|_{-1}^1 = 1^2 - (-1)^2 = 0$.